ENGINEERING CASE LIBRARY

ECL 194A

RADIAL LOAD FLOW PROBLEMS IN DISTRIBUTION CIRCUIT ANALYSIS

Part A

The Problem

This case study covers what is, in my opinion, an extremely interesting application of computers to solve an engineering problem. The problems associated with the distribution function of an electrical utility are often overshadowed by those in the generation and planning areas. This application shows that there are unique problems in distribution which can be solved on a computer and thereby relieve the engineer from relatively tedious jobs. This study is also interesting in that it involves solution of a non-linear problem. It appears that most textbooks only consider circuits which are linear in nature. However, in real life, many problems are non-linear and require special logic in order to arrive at a satisfactory solution.

Francis A. Long
Pennsylvania Power
and Light Company

Written by C. E. Prince, Jr. with the support of the Middle Atlantic Power Research Committee. Published by the Engineering Case Program with support from the National Science Foundation.

INTRODUCTION

One day in December, 1969, Harold E. Taylor, Senior Engineer, Distribution Lines and Substation Department of Division Operations, Pennsylvania Power and Light Company, called Frank Long, who was Senior Programming-Engineering Analyst in the Methods Section of PP&L's Financial Division. "Frank, couldn't we do something to calculate single-phase loads correctly?" asked Taylor.

Following this contact, a few meetings between Frank and Harold showed that what was needed was a more precise way to analyze a distribution circuit and calculate its voltage profile in order to determine the proper size and location of fixed and switched capacitors on PP&L's distribution circuits. A recent study made by the company had shown capacitor usage to be indeed an economical method for controlling voltage at the customer's meter in order to satisfy Pennsylvania Public Utility Commission regulations.

The large, fast digital computers used by PP&L could handle a comprehensive distribution circuit analysis program which would not only help to spot capacitors at proper locations but aid in other ways to guide decisions necessary to control distribution voltages in an economical way. But Frank said, "What I had in mind was not merely a replacement for existing rough ways of calculating these things. We really wanted a method to reduce the manpower required to manage a distribution circuit. It is a non-linear circuit analysis problem and later, when we get the system working we should be able to locate and size capacitors in the optimum manner."

FRANK LONG, JERRY GUTH, AND THE METHODS SECTION OF PENNSYLVANIA POWER AND LIGHT

The Methods Section performs all scientific computer programming for PP&L. Equipped with the IBM 360/65 and 360/50 computers as well as some older, "second generation," computers such as IBM 7074 and 1401, the Methods Section is composed of five engineers and eleven mathematicians,

three of whom are women, who program problems arising in PP&L's work and send the programs into the company's computer center.

Frank Long, 1963 graduate from Northeastern University, has done additional studying both at PP&L and at nearby Lehigh University. After five years experience as engineer in PP&L's System Planning Department, he came to his present position in the Methods Division two years ago. "With system engineering experience and with further work in computer methods, I believe I can guide programming from both the engineer's and mathematician's viewpoints. It's an interesting, creative job, and I enjoy the challenge," Frank remarked.

After the initial discussions with Taylor, Frank roughly estimated the job would take a certain number of man-hours and selected Gerald Guth, a scientific programmer, to work with him. Jerry graduated from Lehigh University in 1966 with a degree in engineering physics and later a master's degree in math. At the time of this distribution circuit assignment, Jerry was working on a master's degree in physics from Drexel Institute of Technology under PP&L's tuition repayment plan. Frank said, "I asked Jerry to work with me on this problem because I felt he was capable and had the time available." Jerry came to the Methods Section in 1968, after working in the Atomic Power Section for two years since joining PP&L.

"The job," Frank said, "was actually not of a top priority nature. We'd had something that worked for some time which we could live with--an approximate way of treating the case. However, we all felt that in the end significant improvements would result."

Frank further explained that Jerry and he had allotted time to work on this job steadily unless urgent jobs came in--and there had been several such instances. "For example," Frank pointed out, "Jerry once had to drop everything in order to check and correct an ASME program which calculated enthalpy from pressures and temperatures. This work was done on a hurry-up basis since our Brunner Island No. 3 generation plant badly needed the correct data."

PENNSYLVANIA POWER AND LIGHT COMPANY

The company serves a 10,000 square mile area in Central-Eastern Pennsylvania. It is an investor-owned public utility with 98,450 shareowners; its headquarters is in Allentown, Pennsylvania. PP&L is part of a large power pool

known as the Pennsylvania-New Jersey-Maryland Interconnection, composed of 13 companies serving an area of 48,700 square miles. PP&L produces electric power for residential, rural, commercial, and industrial users from its five steam, three hydro-electric, and nine combustion-turbine powered generating stations and, in time of heavy demand--in wintertime, at present, due to an increase in number of electrically-heated homes--can also purchase power from neighboring utilities.

In 1959, peak demand was 1292 megawatts; in 1969, 2702 megawatts--more than double. The company management expects the demand to continue increasing, and in fact, lists the following as major problem areas: increased taxes, inflation because of higher costs for construction and operation, increased demand for electricity, environmental protection, and higher standards for electric service reliability.

Figure 1A of Appendix A shows the main PP&L system in 1974. Figure 2A gives further data on PP&L. Figure 3A is a simplified organization chart with the positions of the principals in this case shown.

LEGAL AND PRACTICAL ASPECTS OF THE PROBLEM

The Pennsylvania Public Utility Commission sets operating and finance standards for all investor-owned electric utilities in the state. The regulations specify that voltage (60H_z) at the customer's meter shall not exceed the nominal standard (120 volts for PP&L) by ± 5%, or between 114 and 126 volts, and, further, that the total variation between sunset and 11:00 p.m. shall not exceed 8%. Thus, one objective is to comply with the regulation, but also, as Frank commented, to do so in an economical, reliable manner.

UNDESIRABILITY OF VOLTAGE VARIATION

Frank further pointed out a study done by another PP&L engineer on the effects of voltage variation on customer utilization equipment. This study gave figures to support the common-sense conclusion that the best combination of reliability, lifetime, efficiency, and economy is achieved if electrical equipment is operated at the design voltage.

For example, induction motors overheat or may not start at all, resistance heating devices take longer to heat up on the one hand or shorten in life on the other, and tonal

quality and volume, brilliance, and focus of black-white and color TV sets may be affected. Lumen outputs and theoretical life of incandescent lamps are drastically altered by voltage changes. Figures 4A, 5A, and 6A of Appendix A suggest the importance of maintaining proper voltage.

THE DISTRIBUTION CIRCUIT MANAGEMENT PROBLEM

The distribution circuit, as explained by Frank, consists of the power lines from the substation to the customer's location. "It usually begins with a 66-12KV step-down transformer and is made up of a main three-phase line out to a certain point, with branches of three-phase or single-phase taps from place-to-place," Frank said. (Figure 7A of Appendix A shows an actual lay-out of a circuit.) Wires are strung on poles according to standard techniques, two of which are shown for 12KV lines in Exhibit 1A of Appendix A, page 9A.

The input data for a distribution analysis consists of the KW and KVAR--+ for inductive, - for capacitive--loads on each phase, the phase connection data, distances between nodes, wire sizes, and voltage at the source, or bus. Using the wire data and construction, or pole, type, the resistance and impedances per mile may be calculated (reference 1 in the bibliography).

Figure 8A of Appendix A gives conductor data typically used and Exhibit 2A gives a printout of circuit data for an actual circuit on the PP&L system. With this data, a voltage profile for peak and light load conditions, with and without fixed or switched capacitors on the system, can be plotted to be sure that voltage criteria are maintained at all times.

Capacitor banks, connected at various points, are used to control voltage levels by providing negative KVAR's to loads requiring positive KVAR's. Frank explained, "It's the power factor correction problem, actually. But the circuit equations are non-linear since we assume a constant KVA load--a little bit different from the problems you find in textbooks."

Primary distribution systems should be designed and operated so that transformers can be economically loaded and still provide a customer voltage variation within allowable limits. On the average, with some allowance for overloading the distribution transformer, the voltage drops may be apportioned as follows:

	Peak Load	Light Load
Drop from 66KV to 12KV bus	1%	.5%
Drop in 12KV line	2%	.9%
Drop in distribution transformer	3%	1.3%
Drop in secondaries and services	3%	1.3%
Total	9%	4.0%

According to Frank, the preceeding apportionment does not make efficient use of the system facilities, particularly in the 12KV circuit unless voltage compensation is applied. For example, if limited to a two percent voltage drop on the 12KV circuit, a 10-mile, 3-phase line of No. 2/0 copper conductor, uniformly distributed load among the 3-phases, would be limited to less than 10% of thermal capability.

On the PP&L system the voltage at any given location can be established by the use of one or more of the following alternate methods:

For general level control--a rough adjustment--

1. Taps on the 66-12KV transformers
2. Fixed capacitors on the distribution circuit
3. Taps on the distribution transformers

For voltage compensation--defined as

$$\frac{\text{11 a.m. voltage - 3 a.m. voltage}}{\text{3 a.m. voltage}} \times 100\text{--}$$

1. Variation of the 66KV voltage by varying the generator field strength, and tap changing under load of the 220-66KV transformer in the substations
2. Switched capacitors on the 12KV distribution system
3. Use of voltage regulators.

In 1962 PP&L concluded from a broad study (see reference 3 in the bibliography) that the optimal method for controlling distribution voltage was by the application of fixed and switched capacitors--fixed capacitors would be connected at a given location and left for a longer period of time; for short-term needs, other switched capacitors would automatically switch on during peak load times and later switch off the lines. Application of this pattern is intended, Frank said, to maintain a primary distribution voltage within the limits of 120-126 volts during peak load periods and 119-123 volts during light load periods. These voltages are not the actual primary voltages but the primary voltage referred to secondary.

Frank said that the purchase of variable tap transformers was discontinued in the spring of 1962 because a change in the manufacturer's transformer standards added a premium of $20.00 for taps to the purchase price of a 10 KVA transformer and $37.00 to a 25 KVA transformer. For the PP&L Company, this would have resulted in a total increase of $132,000 for a year's supply of transformers.

Also, the use of voltage regulators on the distribution circuits was undesirable from both the consideration of initial capital outlay and later high maintenance costs.

Frank pointed out that the use of capacitors, properly sized and located, not only promised better economy but also would improve the over-all system power factor and thereby release thermal capability on the entire system from the generators to the distribution transformers at significant savings.

For example, the 1962 study on use of capacitors stated that using $3 million to supply capacitors to the PP&L system would raise the total system power factor at peak load from 80 to 90%, possibly resulting in decreased transmission and distributien losses saving, under optimum conditions, $42 million/year! Actually this large saving estimate was not realistic, but there was no doubt at PP&L of the economic benefits.

CALCULATION OF VOLTAGE PROFILES

Since voltage is the item of interest along the distribution circuit, plots of voltage vs. distance from the 66 KV bus were calculated by PP&L distribution engineers, Frank related, first by hand and later by computerizing the hand method. Using the data from the simple distribution circuit and the phasor relations shown in Figure 10A

of Appendix A, the voltages along the circuit were calculated by Frank and Jerry using the hand, or approximate, method. Results are shown in Figure 11A for the circuit when the capacitors are connected. These numbers and the resulting voltage profile were to be compared with the exact method to be programmed.

Examples of actual voltage profiles calculated by a PP&L engineer are shown in Exhibits 3A and 4A of Appendix A, pages 14A and 15A.

Frank explained that, in brief, the steps commonly used in checking a circuit for proper voltage were:

1. Calculation of a light load voltage profile with no capacitors. If the light load 12KV bus voltage--120 volts used as a base--was below 120 volts, the 66-12KV transformer tap change was calculated to bring the bus to 122 volts.

2. Then, after attaching all fixed capacitors, calculation of a new profile for the three-phase main line. If the lowest voltage was less than 118 volts, the fixed capacitors are determined to raise the lowest point to 119 volts.

3. Repetition of step 2 for peak load conditions. If the lowest voltage were found to be less than 119 volts, additional switched capacitors were added beyond the lowest point toward the end of the line to raise the minimum voltage to 120-122 volts.

4. Repetition of steps 2-3 for single-phase taps.

Other considerations entered into the problem too, but Frank emphasized that the key was an <u>ability to analyze the circuit</u>.

The $V = PR + QX$ relation was admittedly approximate and for single-phase loads on a single-phase circuit an approximation led to use of a multiplier of 4.5 to compute the voltage drop. Also source voltages for the single-phase taps were those obtained from the averages of the three-phase circuit voltages at the point of tap. However, without the computer to solve a large distribution circuit exactly, the hand calculation work was completely out of reason, so the approximate method was obviously the only solution available in prior days.

Frank said, "Traditionally, solution of unbalanced three-phase circuit problems have been done by the method

of symmetrical components. This is impractical by hand here, so approximate methods had to be used. Also, the whole thing depends on information about the circuit--loads, wire sizes, and so forth--which, of course, are known only to a certain accuracy. However, there appeared to be cases in actual practice, according to Harold Taylor, where a more accurate way, made possible by the computer, could be used profitably."

With these considerations in mind, Frank and Jerry began to study the problem in detail. They charged their own and computer time to an assigned account number and eventually Taylor's department will be charged with the development cost.

What would _your_ approach to this problem be?

APPENDIX A

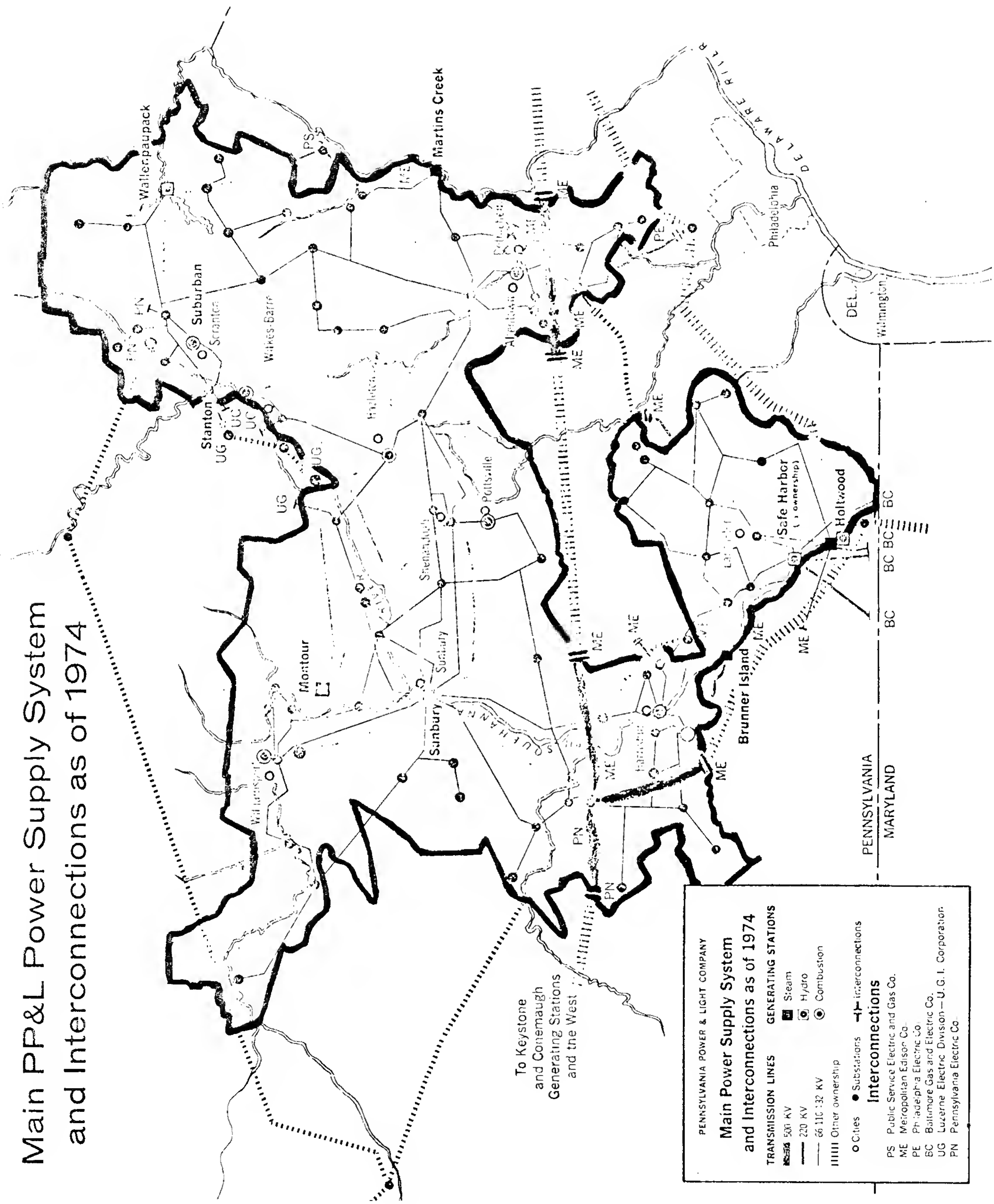
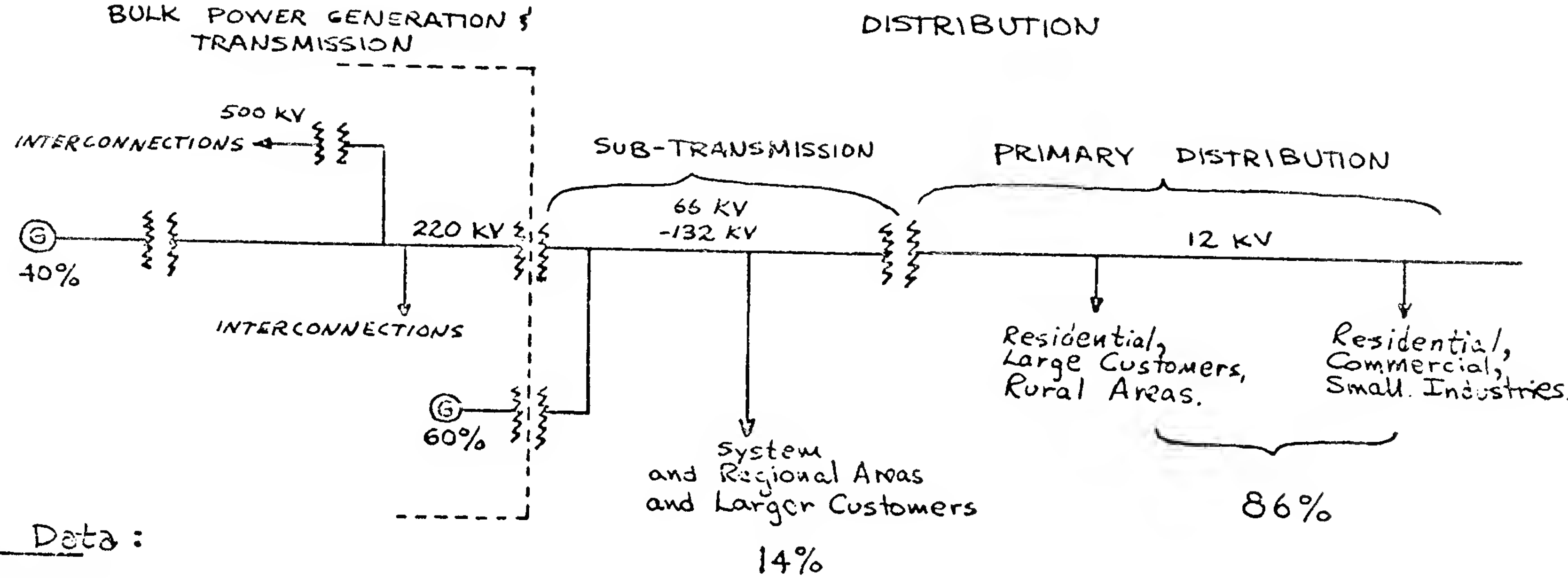

Main PP&L Power Supply System
and Interconnections as of 1974
Wallenpaupack
Martins Creek
Suburban
Scranton
Wilkes-Barre
Stanton
Montour
Sunbury
Pottsville
Shenandoah
Allentown
Philadelphia
Wilmington
DEL.
DELAWARE RIVER
Safe Harbor
Holtwood
Brunner Island
Lancaster
Harrisburg
PENNSYLVANIA
MARYLAND
To Keystone
and Conemaugh
Generating Stations
and the West
PS
ME
PE
BC
UG
PN
PENNSYLVANIA POWER & LIGHT COMPANY
Main Power Supply System
and Interconnections as of 1974
TRANSMISSION LINES
500 KV
220 KV
66 110 132 KV
Other ownership
GENERATING STATIONS
Steam
Hydro
Combustion
Cities
Substations
Interconnections
Interconnections
PS Public Service Electric and Gas Co.
ME Metropolitan Edison Co.
PE Philadelphia Electric Co.
BC Baltimore Gas and Electric Co.
UG Luzerne Electric Division—U.G.I. Corporation
PN Pennsylvania Electric Co.

Figure 1A

PP&L SYSTEM



1969 Data:

Power Capability: 3124 MW
Peak Demand: 2702 MW
No. of Customers: 818,500 in 10,000 mi.^{2} area Central - Eastern Pennsylvania
(6 cities > 50,000 (max. 115,000), 57 towns > 5000)

No. of Employees: 6,238
Total Revenue: $223,388,000
Total KW-hr. Sold: $13,531 \times 10^6$
Avg. price / KW-hr.: 1.61¢
Std. Distribution Ckt.: No. 2/0 Copper Equivalent 12-KV circuit, multi-grounded neutral, transformer connected phase-to-neutral. (Lines loaded to 7500 KVA maximum and 9000 KVA in emergencies.)

Figure 2A

PP&L Organization Chart (simplified)

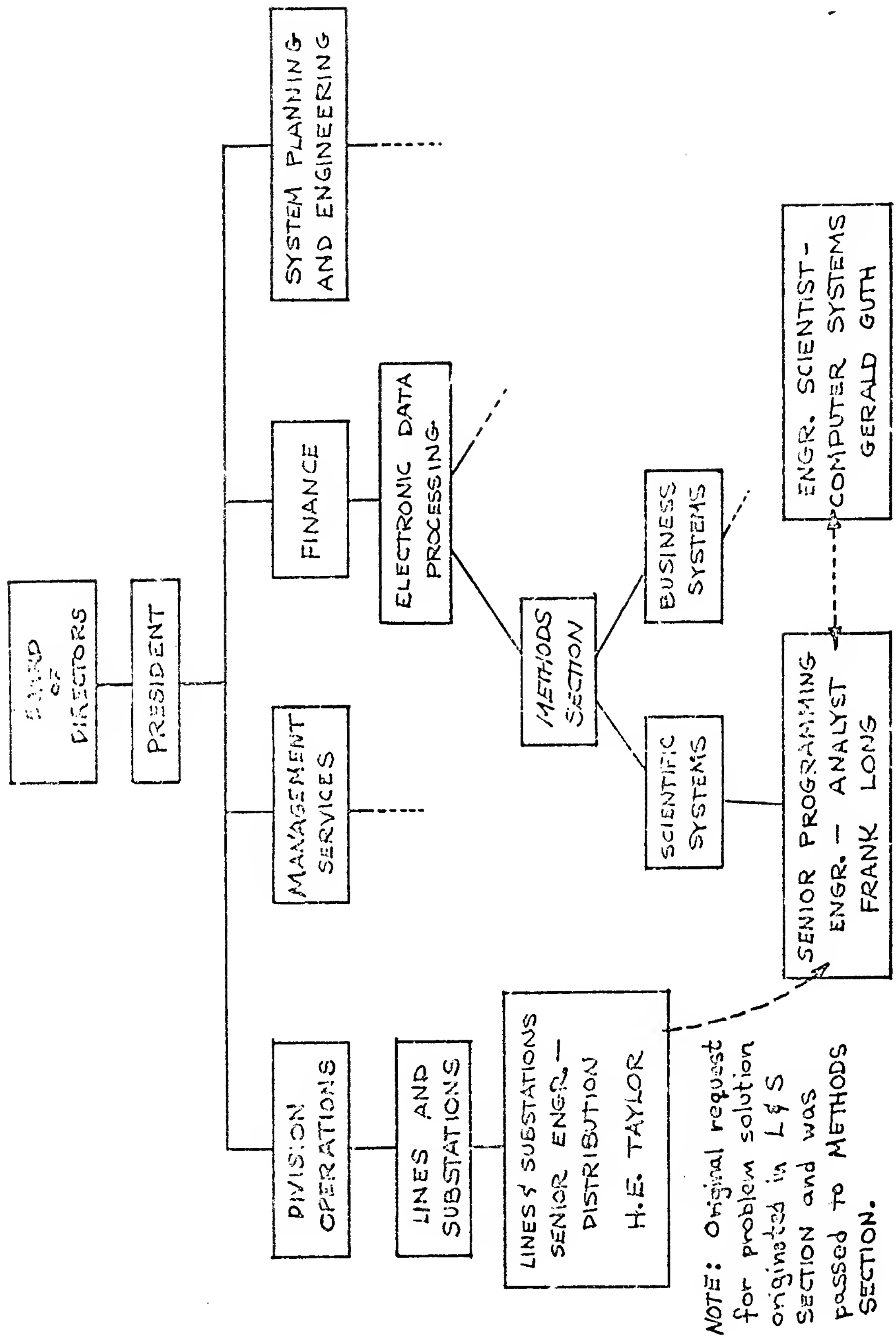
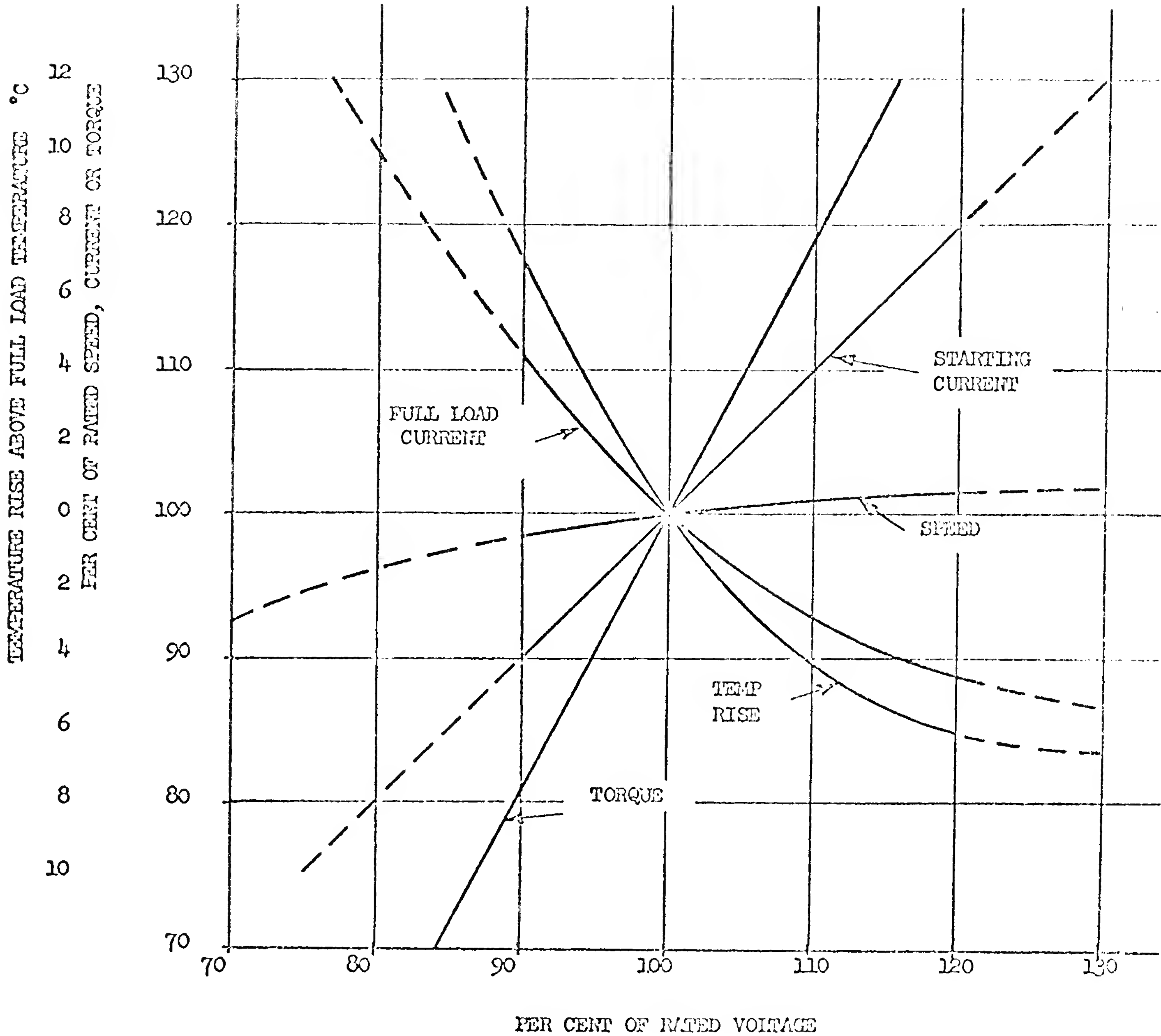


Figure 3A

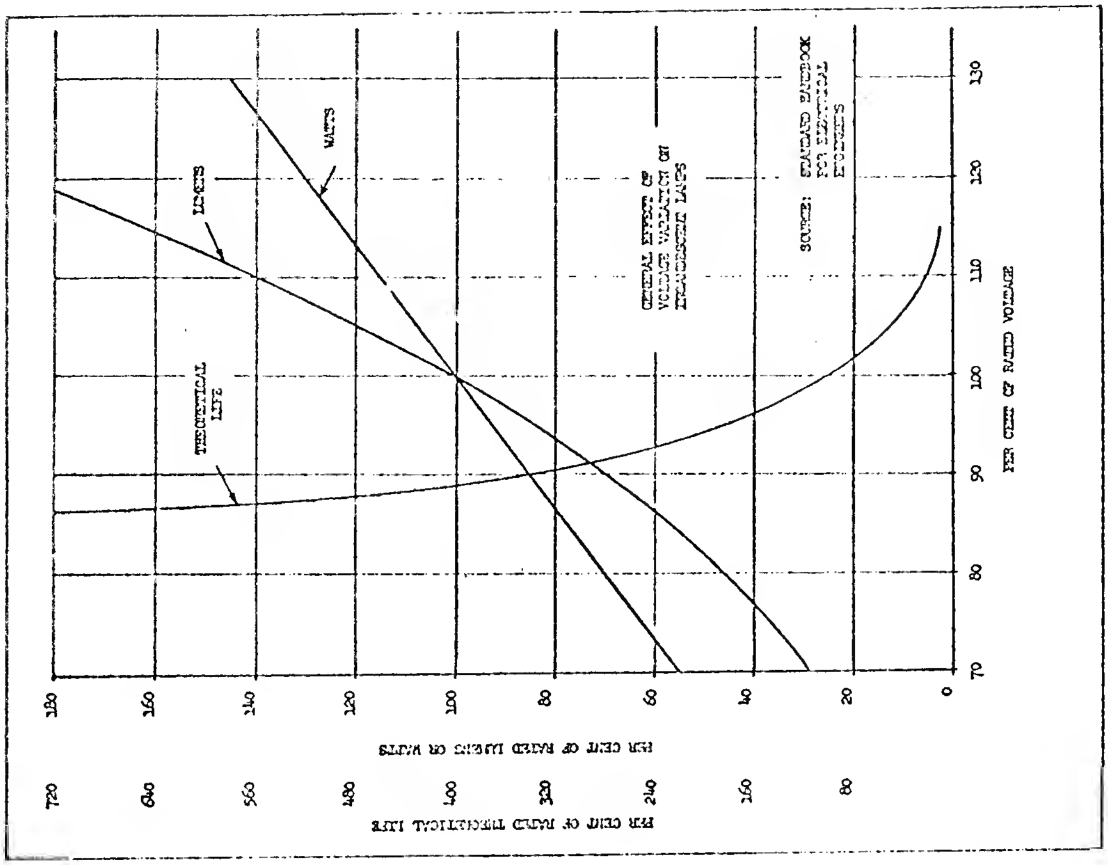


GENERAL EFFECT OF VOLTAGE VARIATION
ON INDUCTION MOTOR CHARACTERISTICS
CONSTANT TORQUE LOAD

NOTE: THESE EFFECTS WILL VARY SOMEWHAT FOR SPECIFIC RATINGS

Figure 4A

SOURCE: STANDARD HANDBOOK FOR ELECTRICAL ENGINEERS


GENERAL EFFECT OF VOLTAGE VARIATION ON INCANDESCENT LAMPS
SOURCE: STANDARD HANDBOOK FOR ELECTRICAL ENGINEERS
LUMENS
WATTS
THEORETICAL LIFE
PER CENT OF RATED VOLTAGE
PER CENT OF RATED LUMENS OR WATTS
PER CENT OF RATED THEORETICAL LIFE
70
80
90
100
110
120
130
0
20
40
60
80
100
120
140
160
180
160
240
320
400
480
560
640
720

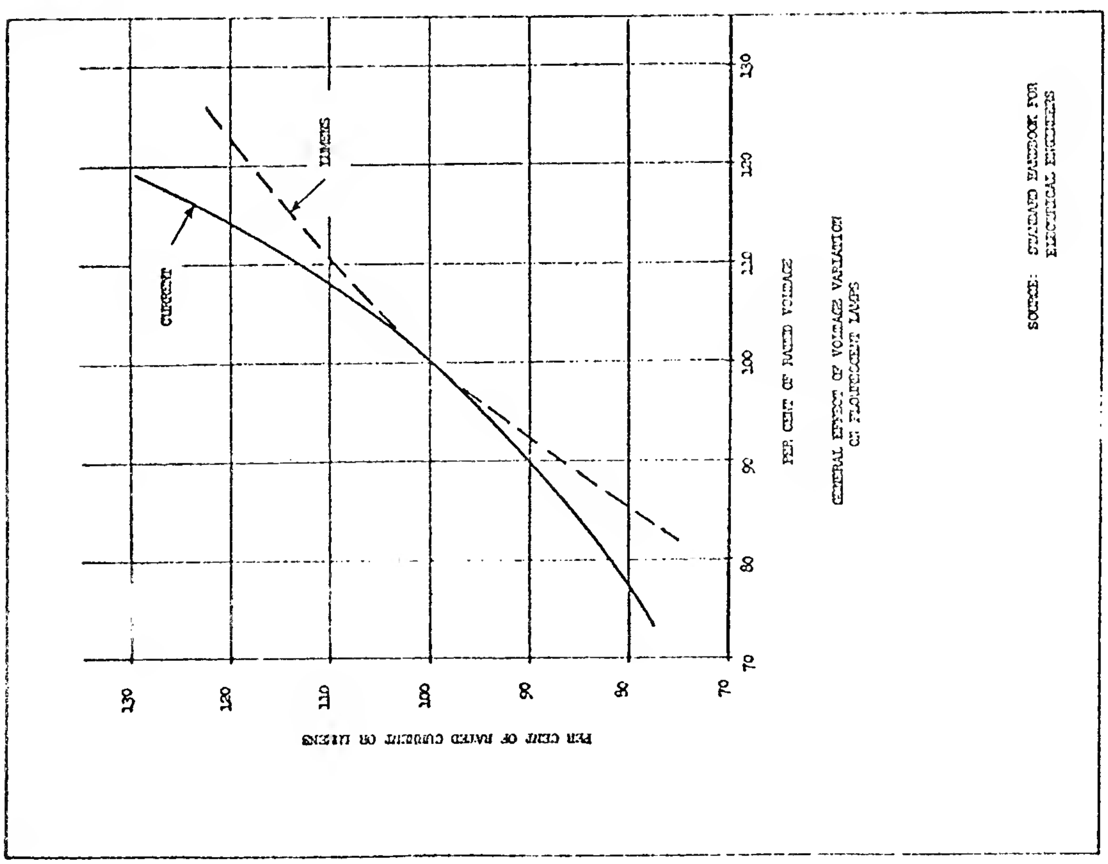

GENERAL EFFECT OF VOLTAGE VARIATION ON FLUORESCENT LAMPS
SOURCE: STANDARD HANDBOOK FOR ELECTRICAL ENGINEERS
CURRENT
LUMENS
PER CENT OF RATED VOLTAGE
PER CENT OF RATED CURRENT OR LUMENS
70
80
90
100
110
120
130

Figure 5A

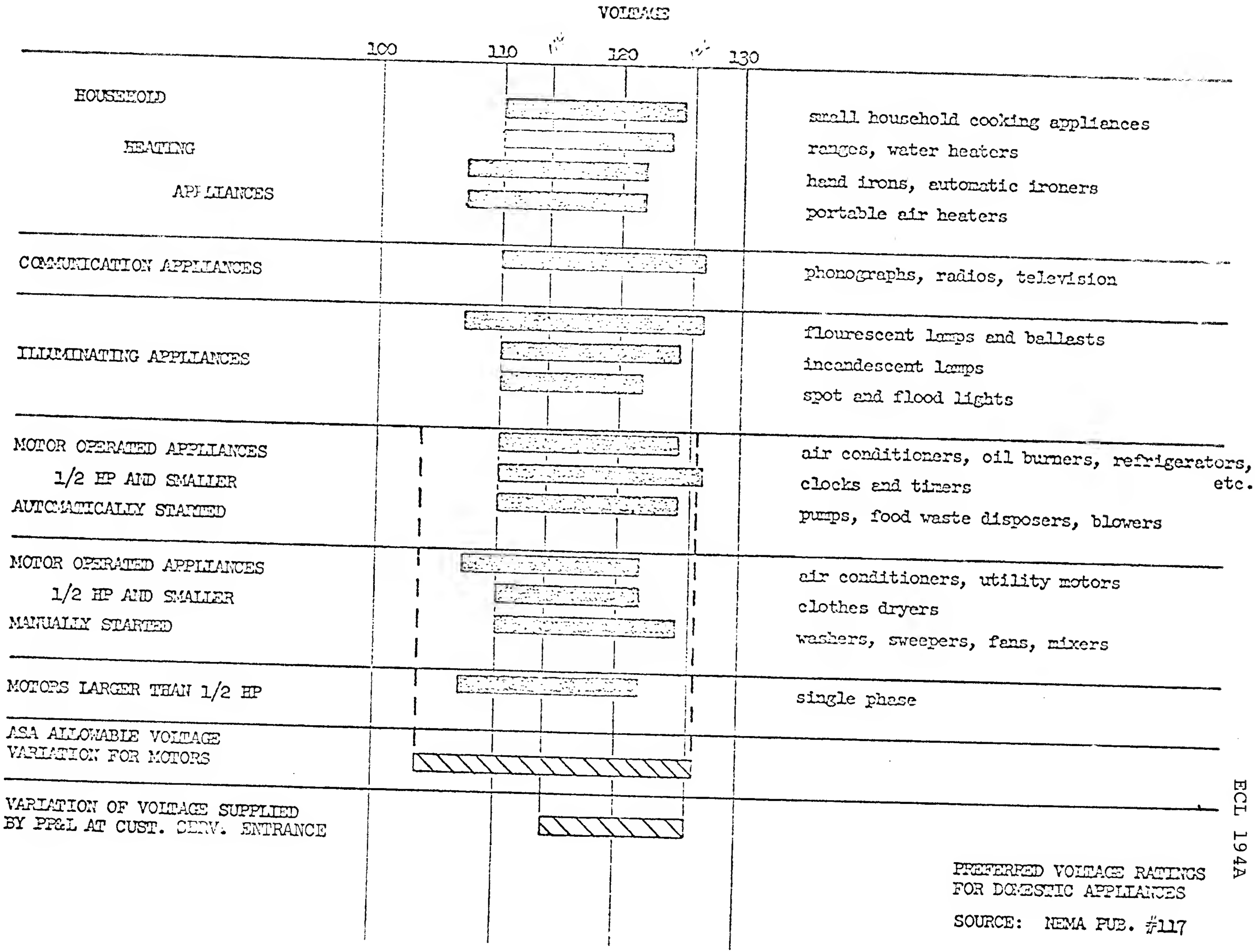

VOLTAGE
100
110
120
130
HOUSEHOLD
HEATING
APPLIANCES
small household cooking appliances
ranges, water heaters
hand irons, automatic ironers
portable air heaters
COMMUNICATION APPLIANCES
phonographs, radios, television
ILLUMINATING APPLIANCES
flourescent lamps and ballasts
incandescent lamps
spot and flood lights
MOTOR OPERATED APPLIANCES
1/2 HP AND SMALLER
AUTOMATICALLY STARTED
air conditioners, oil burners, refrigerators, etc.
clocks and timers
pumps, food waste disposers, blowers
MOTOR OPERATED APPLIANCES
1/2 HP AND SMALLER
MANUALLY STARTED
air conditioners, utility motors
clothes dryers
washers, sweepers, fans, mixers
MOTORS LARGER THAN 1/2 HP
single phase
ASA ALLOWABLE VOLTAGE
VARIATION FOR MOTORS
VARIATION OF VOLTAGE SUPPLIED
BY PP&L AT CUST. SERV. ENTRANCE
PREFERRED VOLTAGE RATINGS
FOR DOMESTIC APPLIANCES
SOURCE: NEMA PUB. #117

Figure 6A

DISTRIBUTION CIRCUIT LAYOUT

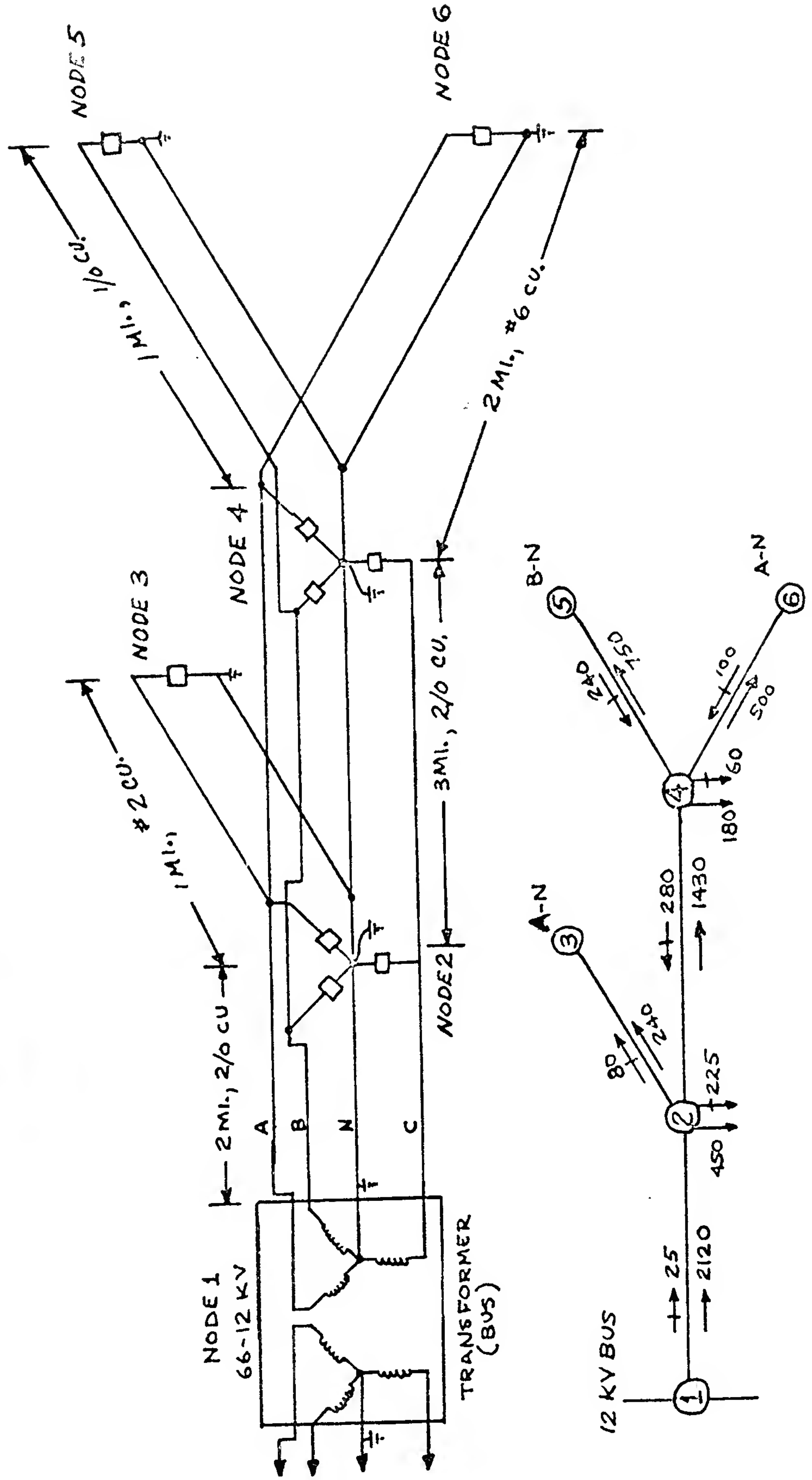


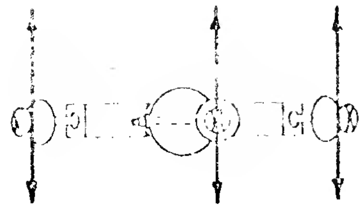
Figure 7A

Distribution Circuit Data

Load Data:

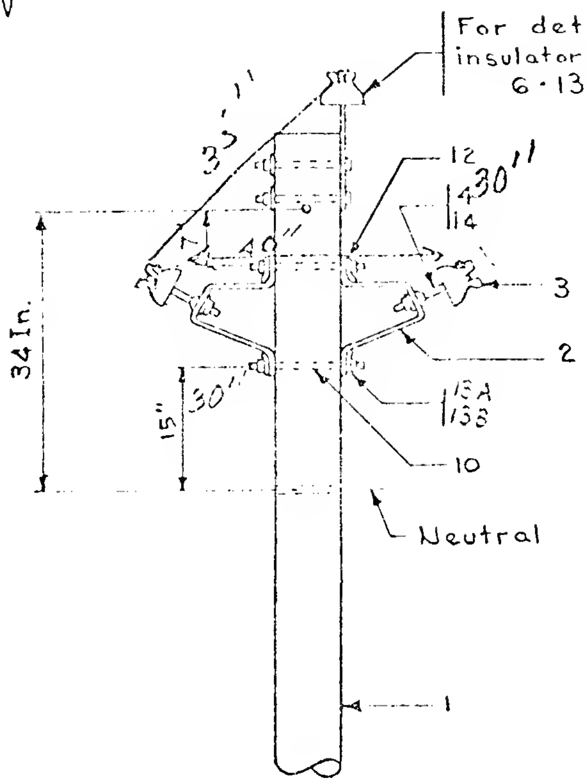
NODE	$PL_{(KW)}$	$QL_{(KVAR)}$ +	$Qc_{(KVAR)}$ -	PL	QL	Qc	PL	QL	Qc
1	0	0	0	0	0	0	0	0	0
2	150	75	0	150	75	0	150	75	0
3	240	80	0	0	0	0	0	0	0
4	60	20	0	60	20	0	60	20	0
5	0	0	0	750	60	300	0	0	0
6	500	100	200	0	0	0	0	0	0

Line	Phase	Miles	Type	R(Ω)	X(Ω)
1-2	ABC	2.0	2/0cu	.062	.092
2-3	A	1.0	#2 cu	.062	.049
2-4	ABC	3.0	2/0cu	.093	.138
4-5	B	1.0	1/0cu	.039	.047
4-6	A	2.0	#6 cu	.300	.104

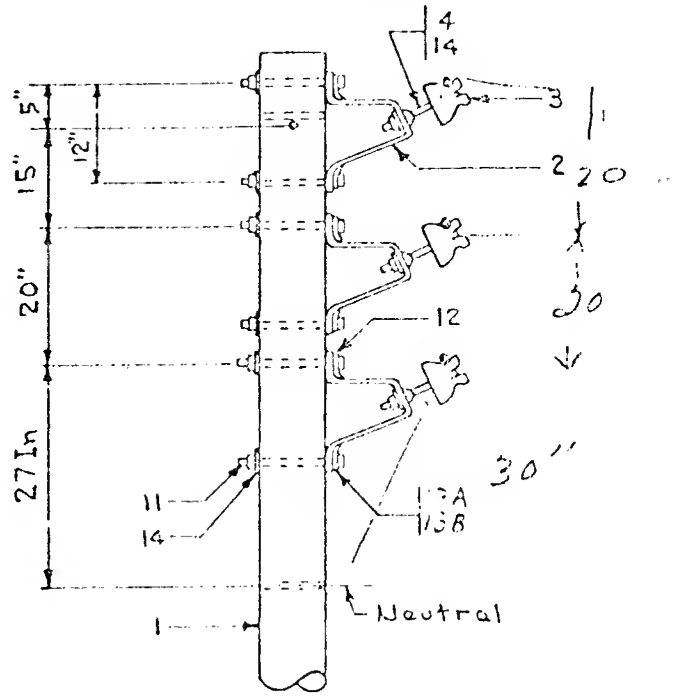


GMD = $\sqrt{}$

GMD = $\sqrt[3]{1.75 \times 1.75 \times 3.5}$
= 2.203'



METHOD 1



METHOD 2

NOTE:
WHEN REQUIRED BY OVERHANGING TREES, A 3/8" H.S. OR 1/2" H.S. OVERHEAD SHIELD WIRE, SPECIFIED BY ENGINEER, SHALL BE INSTALLED AT TOP OF POLE PER 6-23-38.

For B/M see 6-13-60

ORIG. SPONSOR	J.W.B. Rut	CLOSE-SPACED PRIMARY CONDUCTOR PLASTIC BRACKET - THREE PHASE STRAIGHT LINE CONSTRUCTION 12 KV AND BELOW ANGLES 0° TO 3°	PENNA. PWR. & LGT. CO. ALLENTOWN, PA.	
ORIG. T.&D. ENGR.	(W.A.)			
REV.-SPONSOR	J.W.G.		ORIG. APPR. DATE	9-13-65
REV.-T.&D. ENGR.	R.P. Licini		REV.-APPR. DATE	

COPPER CONDUCTOR VOLTAGE DROP TABLE

ECL 194A

KVA MILES FOR 1% VOLTAGE DROP - BALANCED 3 PHASE CIRCUIT

12.47 KV EQUIV. SPACING 4'-8" GMD

P.F.	#6	#4	#2	#1	1/0	2/0	3/0	4/0
100	663	1030	1610	2040	2560	3240	4070	5130
95	630	926	1350	1630	1940	2300	2700	3120
90	630	910	1300	1540	1800	2090	2410	2730
85	642	910	1280	1500	1720	1980	2260	2510
80	658	926	1270	1470	1680	1910	2160	2380
70	700	962	1280	1460	1640	1830	2040	2220
60	752	1010	1310	1470	1630	1800	1980	2130
50	827	1080	1360	1510	1660	1800	1970	2090
40	925	1170	1440	1570	1700	1830	1980	2080
35	980	1230	1480	1610	1730	1850	1990	2080
30	1050	1290	1530	1650	1760	1870	2010	2090

% IMPEDANCE PER MILE ON 10 MVA BASE AT 12.47 KV

	#6	#4	#2	#1	1/0	2/0	3/0	4/0
R	15	9.8	6.2	4.9	3.9	3.1	2.5	1.9
X	5.2	5.0	4.9	4.8	4.7	4.6	4.4	4.4
Z	16	11	7.9	6.9	6.1	5.5	5.1	4.8

KVA OF CAPACITORS FOR 1% RISE PER MILE OF LINE

#6	#4	#2	#1	1/0	2/0	3/0	4/0
1920	2000	2040	2080	2130	2175	2270	2270

KVA MILES FOR 1% VOLTAGE DROP - 4.16 KV EQUIV. SPACING 2'-10"

P.F.	#6	#4	#2	#1	1/0	2/0	3/0	4/0
100	72	114	180	226	285	362	453	572
95	69	104	153	185	220	262	309	361
90	70	103	148	175	206	241	280	318
85	71	104	146	171	199	230	263	296
80	73	105	146	169	195	222	254	281
70	78	109	148	169	191	215	241	264
60	85	116	152	171	192	212	236	255
50	94	125	159	177	196	214	236	250
40	105	136	169	185	202	218	238	250
35	112	143	174	190	206	220	239	251
30	120	151	181	196	210	225	242	252

% IMPEDANCE PER MILE ON 10 MVA BASE AT 4.16 KV

	#6	#4	#2	#1	1/0	2/0	3/0	4/0
R	139	88	56	44	35	28	22	18
X	43	42	40	40	39	38	36	36
Z	145	97	69	59	52	48	43	40

KVA OF CAPACITORS FOR 1% RISE PER MILE OF LINE

#6	#4	#2	#1	1/0	2/0	3/0	4/0
232	238	250	250	256	263	278	278

Figure 8A

BLOOMSBURG-MILLVILLE 12KV CASE NO.10
UNITY SINGLE PHASE TAP
360 TEST SEPT.68

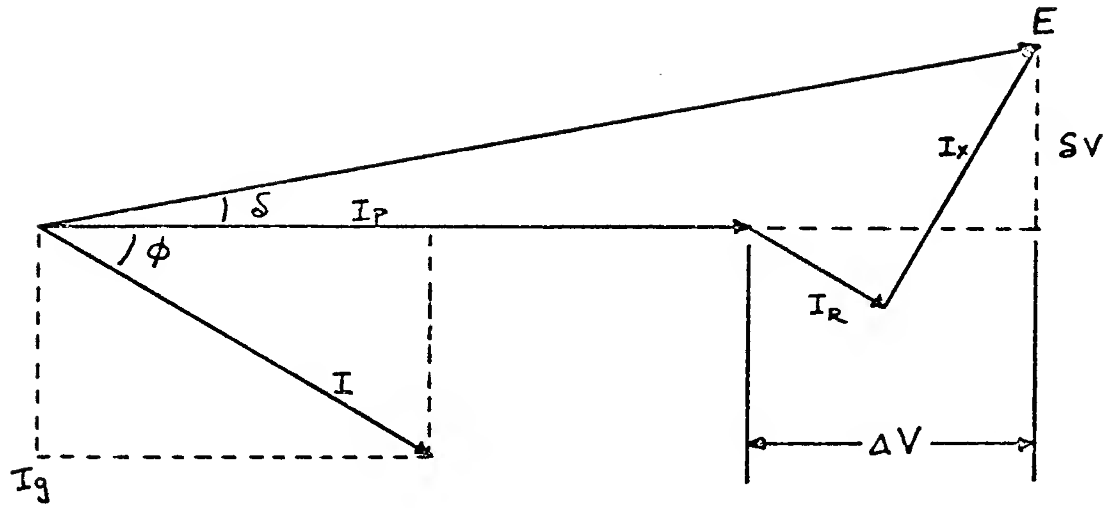
BUS VOLTAGE OF 123.0 AT PEAK LOAD POWER FACTOR OF -0.95 BASED ON EXISTING LINE CONDITIONS
BUS VOLTAGE OF 120.5 AT LIGHT LOAD POWER FACTOR OF -0.97 BASED ON EXISTING LINE CONDITIONS

BUS REACTANCE 0.04
TOTAL PEAK LOAD 5044 KVA
BUS VOLTAGE IS 12KV

INPUT DATA FOR DISTRIBUTION CIRCUIT ANALYSIS

LINE SEGMENT NAME	FROM NODE	TO NODE	CHECK VOLTS	PEAK LOAD KVA	LLKW % OF PLKW	CAPACITORS SW. KVAR	CAPACITORS FIX KVAR	P	WIRE DATA MILE	WIRE DATA SIZE	WIRE DATA MAT.	LINE DATA SEGMENT %R	LINE DATA SEGMENT %X	LINE DATA CUMULATIVE %R	LINE DATA CUMULATIVE %X
SYSTEM-BUS	0	BUS				8730	4440					0.8	4.0	0.8	4.0
BUS-NUMIDIA	1	2		1800	35	1610	0	3	0.1	500	CU	0.1	0.4	0.9	4.4
NUMIDIA-MORDAN	2	3		560	35	0	0	3	4.9	1	CU	24.1	23.5	25.0	27.9
MORDAN-MILL	3	5	118.5	1550	35	1200	600	3	4.5	1	CU	22.1	21.6	47.1	49.6
MILL-IOLA	5	7		660	35	0	0	3	2.5	4/0	ACSR	9.5	12.3	56.6	61.9
IOLA-LOC.9	7	9		51	35	0	0	1	1.4	6A	CU-W	22.0	7.5	78.6	69.4
LOC.9-LOC.11	9	11		33	35	75	0	1	2.2	6A	CU-W	34.5	11.8	113.1	81.2
LOC.11-REGULATE	11	13		0	0	0	0	1	0.1	4/0	CU	0.2	0.4	113.3	81.7
REG.-LOC.15	13	15		49	35	0	0	1	3.1	6A	CU-W	48.6	16.6	162.0	98.3
LOC.15-UNITY	15	17		104	35	0	0	1	0.6	2	ACSR	6.5	3.3	168.5	101.6
UNITY-END	17	19		72	35	0	0	1	0.4	2	ACSR	4.3	2.2	172.8	103.8
IOLA-PINE SUMM.	7	30		130	35	90	0	1	4.2	6A	CU-W	65.9	22.6	122.5	84.5
PINE-LOC.34	30	34		0	0	0	0	1	0.1	4/0	CU	0.2	0.4	122.7	84.9
LOC.34-LOC.38	34	38		35	35	100	0	1	2.8	2	ACSR	30.4	15.3	153.2	100.2

Figure 9A



Phasor Diagram

$$E^2 = (V + \Delta V)^2 + (\delta V)^2$$

$$= (V + RI\cos\phi + XI\sin\phi)^2$$

$$+ \left(\frac{XP}{V} - \frac{RQ}{V}\right)^2$$

Assuming $\delta V << V + \Delta V$

$$E = V + \frac{PR + QX}{V}$$

$$E - V = \Delta V = \frac{PR + QX}{V}$$

Also assume $V \simeq E \simeq 1$ p.u.

$\Delta V_{p.u.} = PR + QX$ if P, R, Q, X are p.u. values.

Figure 10A

Approximate Hand Calculations for Figure 7A Circuit

P_{12} = 2120 + j 25
P_{23} = 240 + j 80
P_{24} = 1430 - j280
P_{45} = 750 - j240
P_{46} = 500 - j100

(KW and KVAR)

12,000 or 120 v = 1 p.u.
All R's and X's on 10,000 KVA base

Let Bus voltage = 126 volts = 1.05 p.u.

$\Delta V_{12} = P_{12}R_{12} + Q$
$= (.2120)(.062) + (.0025)(.092)$
$= .0132 + .00023 = .0134$
$E_2 = 1.05 - .0134 = 1.0366$ (124 volts)

$\Delta_{23} = (.0240 \times .062 + .008 \times .049) \times 4.5$
$= (.00149 + .000392)4.5 = (.00189)4.5 = .0085$
$E_3 = 1.0366 - .0085 = 1.028$ (123 volts)

$\Delta V_{24} = (.1430 \times .093 - .0280 \times .138)$
$= .0133 - .0037 = .0096$
$E_4 = 1.0366 - .0096 = 1.027$ (123 volts)

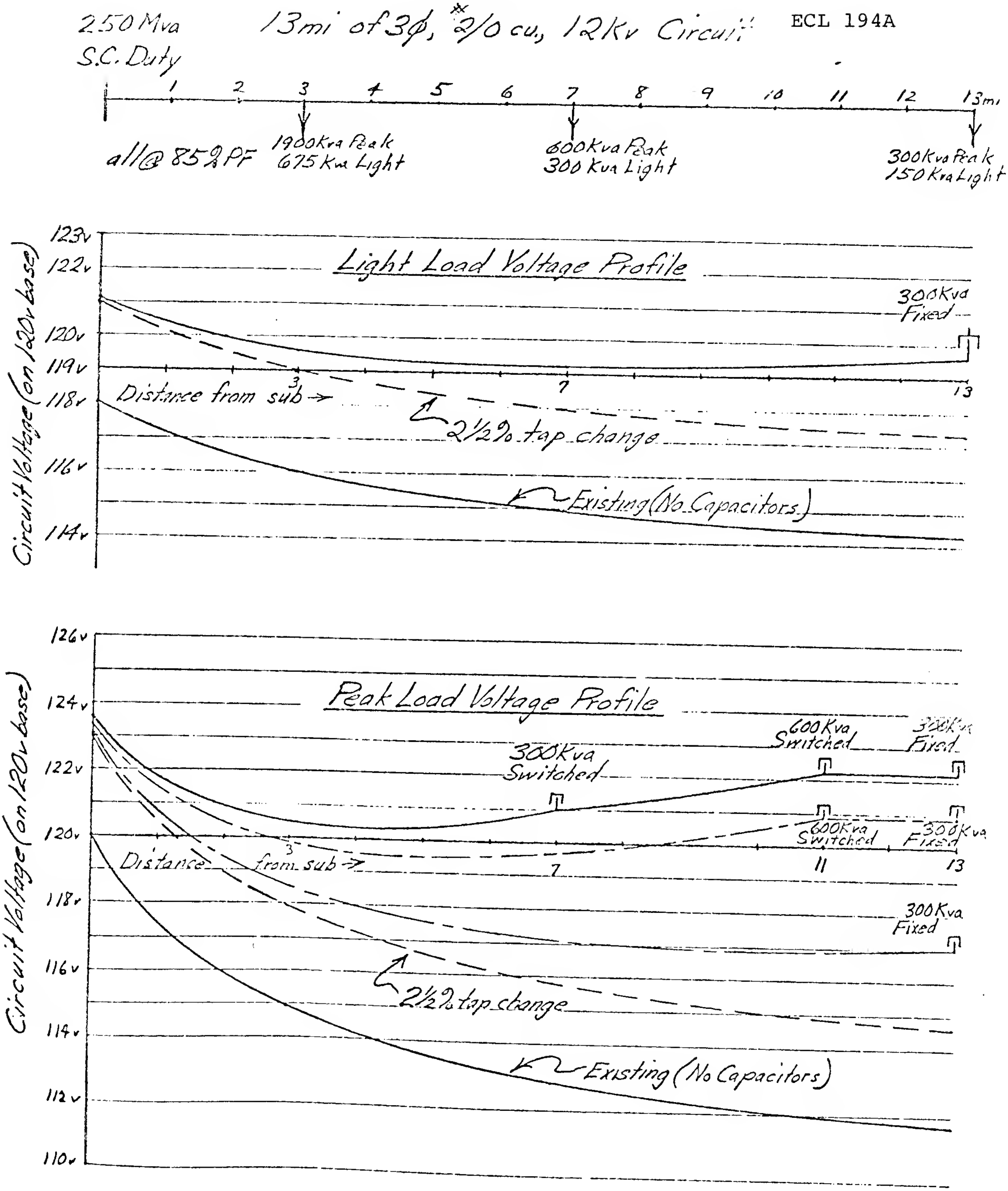
$\Delta V_{45} = (.0750 \times .039 - .0240 \times .047)4.5$
$= (.00292 - .00113) \times 4.5 = (.00179) \times 4.5 = .00805$
$E_5 = 1.027 - .00805 = 1.018$ (122 volts)

$\Delta V_{46} = (.0500 \times .300 - .0100 \times .047) \times 4.5$
$= (.0150 - .00047)4.5 = (.0145) \times 4.5 = .0652$
$E_6 = 1.027 - .0652 = 0.962$ (115 volts)

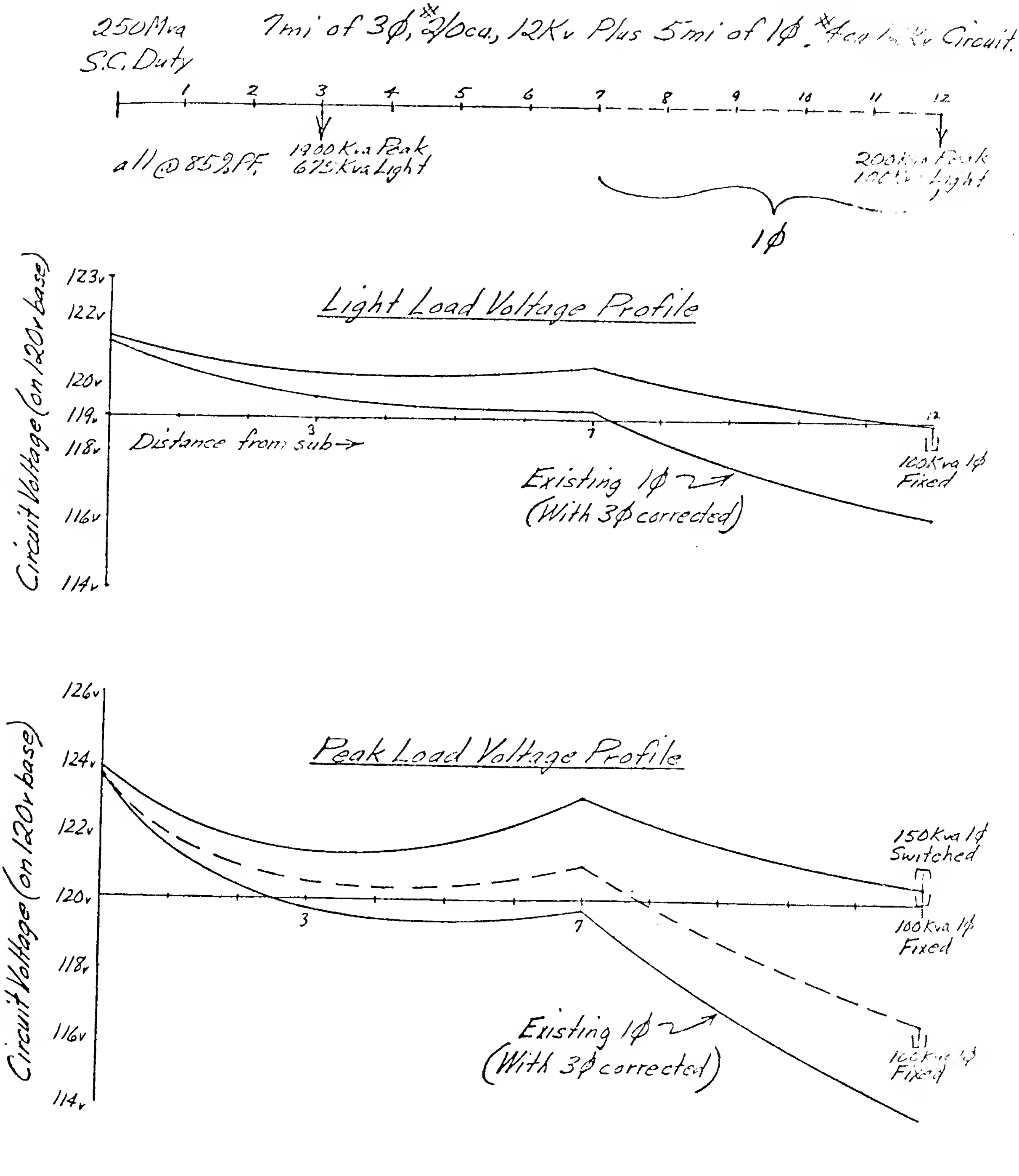
Figure 11A

EXHIBIT 3A

ECL 194A



Voltage Profile for 3φ, 12Kv Circuit Illustrating Voltage Control Philosophy



Voltage Profile for 1φ, 12Kv Circuit Illustrating Voltage Control Philosophy

ENGINEERING CASE LIBRARY

ECL 194B

RADIAL LOAD FLOW PROBLEMS IN DISTRIBUTION CIRCUIT ANALYSIS

Part B

The Engineering Solution

CONSIDERATION OF ALTERNATIVE SOLUTIONS AND APPROACHES

"I felt the most logical thing to do was to go to the literature, and I found most of the really useful stuff in the IEEE *Transactions on Power Apparatus and Systems*," Frank related.

From the beginning, both Frank and Jerry realized the problem was a non-linear circuit analysis problem, as well as one involving an unbalanced three-phase system. The PP&L policy to control voltages by capacitors was firmly established as economically secure; therefore Frank felt no inclination to do other than search the literature for radial load flow calculations on unbalanced three-phase circuits.

"There wasn't too much in the literature on distribution circuits. However, I found an article by a group at Baltimore Gas and Electric which looked good," Frank added. (See reference 2 in the bibliography.) Frank obtained and studied this article and then gave it to Jerry to study and begin writing a suitable computer program.

"We might have obtained the program they used, but much time had passed, computer languages had changed--we program mostly in PL/1 here at PP&L--and it is always difficult to decipher someone else's program," Frank explained. He added, "Actually, the program logic is not so difficult."

According to Frank, the *basic computation scheme* is this:

1. Load admittances are computed using load data and *assuming* a voltage at every load equal to the 12-KV bus voltage.

2. These load Y's and line Z's being used, the circuit problem is solved *once* using as a source voltage the bus voltage, collapsing nodes and wires to form a new node. Nodes farther out from the source bus are collapsed step-by-step back toward that bus. Finally, the circuit is expanded out again to find new values for load voltages.

3. The load voltages which result are then used along with the constant load KW and KVAR to re-compute a new load admittance. Then step 2 is repeated.

4. This iterative process, alternately collapsing and expanding the circuit, is continued until the load voltages calculated in an iteration change less than some specified tolerance away from the immediately prior iteration.

The problem is one in steady-state AC circuit analysis, but with inputs of constant load power, line impedances, and bus voltage, the equations are non-linear and various methods could be used, among them this iterative technique.

Frank further looked over books on circuit analysis for review purposes. Standard load flow techniques such as the Gauss-Seidel or Newton-Raphson methods were discarded as these do not exploit the presence of a radial network so that a special radial load flow program could run faster.

"I gave this paper by the Baltimore group to Jerry, who I felt could do a good job of programming the problem," Frank said.

WRITING THE COMPUTER PROGRAM

Jerry related, "Well, the first thing I did was to read the paper by the Baltimore Gas and Electric group. I had to get a complete understanding of the equations used. Not everything in the way of needed background was in that paper so I had to check some of the paper's references, too. You have to be correct in all details if you're going to put the problem on a computer and get correct answers. So I had to spend some time studying basic circuit theory."

A production program such as this one was to be would not be written by engineers in other parts of the company. Company policy was to assign the problem to a professional programmer, who could write a more efficient, more flexible program, it was thought.

Jerry said that since he was not an electrical engineer and this was really his first electrical engineering problem, he had to spend some time building up his own background. Understanding the subject of line impedance calculations and the effect of multi-grounded neutrals with earth returns was the hardest part. Momentary confusion was caused by mis-numbered figures in the *Transactions* paper, too.

The CPS (Conversational Programming System) available at PP&L was used by Jerry to begin programming after the general theory of the equations to be used was understood. The CPS enabled Jerry to write the initial program statements in the direct interactive mode, by sitting at a keyboard and getting instant acceptance or rejection of program statements typed in.

"I followed my usual practice of breaking the problem down into what seem to be major parts. Then I consider programming each part in turn and assure myself it is technically and logically correct, has available all its necessary input data, and is really computable. I do this before I try to put everything together in one package," Jerry said.

According to Jerry, the reason for this procedure is not only that these sub-units are sometimes best put into subroutines, but also he has found by his own experience, and that of others, that there is a danger in writing large programs of spending too much time on one or several parts and then finding out much later that a latter segment cannot be written or can be written only by radically changing all prior work.

"Actually," Jerry added, "I didn't even write a flow chart for this problem, except maybe mentally. It isn't that big a problem." (The brief flow chart which Jerry later came up with and his own explanation of the procedure of computation are in Exhibit 1B of Appendix B, pages 1B-4B.)

Jerry chose to use nodal equations for his basis, and let the grounded neutral terminal at the 12KV,Y– connected secondary be the over-all distribution circuit reference point. All voltages are calculated with respect to that point.

Although an iterative procedure has within it the possibility of converging to an invalid solution, Jerry felt it was unlikely in this case since starting values of load voltages are really so close, numerically speaking, to the true voltages.

PROGRAM DETAILS AND LANGUAGE

The program was written in PL/1 language. Frank explained this was the current language for all programs to be used internally by PP&L. "PL/1 is definitely user-oriented, and that's why we use it, but actually if speed

is all-important, Fortran is better," Jerry said. He added that Fortran IV is the language used if programs are to be widely circulated. (See Exhibit 2B beginning on page 8B in Appendix B for a listing in PL/1.)

Jerry also had to develop a node-numbering scheme so that a node closer to the source would not be "collapsed" too early. An end-order binary tree data structure traverse scheme followed by a pre-order traverse was the result.

Although variables and computations can be declared and done in the TYPE COMPLEX mode, this was not done here. Frank explained that breaking down the work into real and imaginary parts and writing the program that way saves computer time in the long run. "The complex routines aren't 'in-line' functions so they have to be called up every time they're needed. Programming effort is saved, but computer time is lost," Frank pointed out, "and since the program is to be used over and over again, we feel it better to do things this way." Double precision was not used since the input data was not actually precise enough, time of computation would go up unnecessarily, and round-off error was not expected to be a great problem.

The major numerical method used is Gaussian elimination to solve a system of eight simultaneous linear equations, as Jerry points out in Appendix B, Exhibit 1B.

The conversion of the admittances at a node and the line admittances to that node into driving point admittances at the adjacent node nearest the source bus are done by means of an algorithm mentioned by Jerry and included in Supplement 1B, page 5B. The iterative technique is also stated there.

PROGRAM CHECKING AND RECONSIDERATION

Jerry said, "There were, of course, many minor alterations as I went through the task. After it was done, I felt the basic computational part could be checked with a problem I could solve easily by hand. I tried to do this and got some very peculiar answers."

This trouble caused much "head-scratching." He could find no mistakes in his logic. Repeated trials on problems with known answers gave negative results!

The matrix equation on page 2B of Appendix B is YE = 1 where Y is a 4x4 matrix, E and I both 4-element column vectors, and all elements are complex numbers. Jerry had written

$$(Y_R + j\ Y_I)\cdot(E_R + j\ E_I) = I_R + j\ I_I$$

and multiplying, separating real and imaginary parts,

$$Y_R E_R - Y_I E_I = I_R \tag{1}$$

$$Y_R E_I + Y_I E_R = I_I \tag{2}$$

Multiplying (1) by Y_R, (2) by Y_I, and adding, Jerry had written

$$(Y_R^2 + Y_I^2)\ E_R = Y_R I_R + Y_I I_I$$

and next, (1) by Y_I, (2) by Y_R, and subtracting,

$$(Y_R^2 + Y_I^2)\ E_I = Y_R I_I - Y_I I_R.$$

Thus the real and imaginary parts of the voltage phasor could be computed separately, using 4x4 matrices, saving computer time.

"I had originally scribbled all this down on a yellow, coffee-stained piece of paper and was quite pleased to find a way to uncouple the calculations for the real and imaginary parts of the voltages," Jerry related.

"However," Jerry said, "I finally began to doubt my math rather than the program logic and saw I had done this separation by assuming that matrix multiplication was commutative! After seeing this, I corrected the program and answers came out all right. It's easy to get locked-in mentally on some things."

A main check on the correctness of the computations now is a check on the power balances, real (P) and reactive (Q). All P's and Q's are computed and found to check with that supplied to the network.

FUTURE COMPLETION OF THE PROGRAM AND USE IN PRACTICE

Jerry pointed out that the program is not totally completed yet. Remaining are input-output procedures. Frank also added, "We want to get to the point where off-nominal transformers can be included. And then, of course, now that the circuits problem is solved, we want to study the problem of placing capacitors at optimum locations, and this is a problem in linear programming which remains to be done."

In actual use, data cards with distribution circuit loads and other essential data will be maintained and when analyses are required by distribution engineers, the data can be simply fed into the computer and the program called. Frank said he hoped the various distribution circuit data could eventually be stored on disks to eliminate use of card input.

"It's true that in many instances the former way of calculation is accurate enough, and there's always the problem of doubt on the accuracy of basic input data. But with this program, we've eliminated one more possible area of uncertainty from the problem of managing a distribution circuit," Frank said.

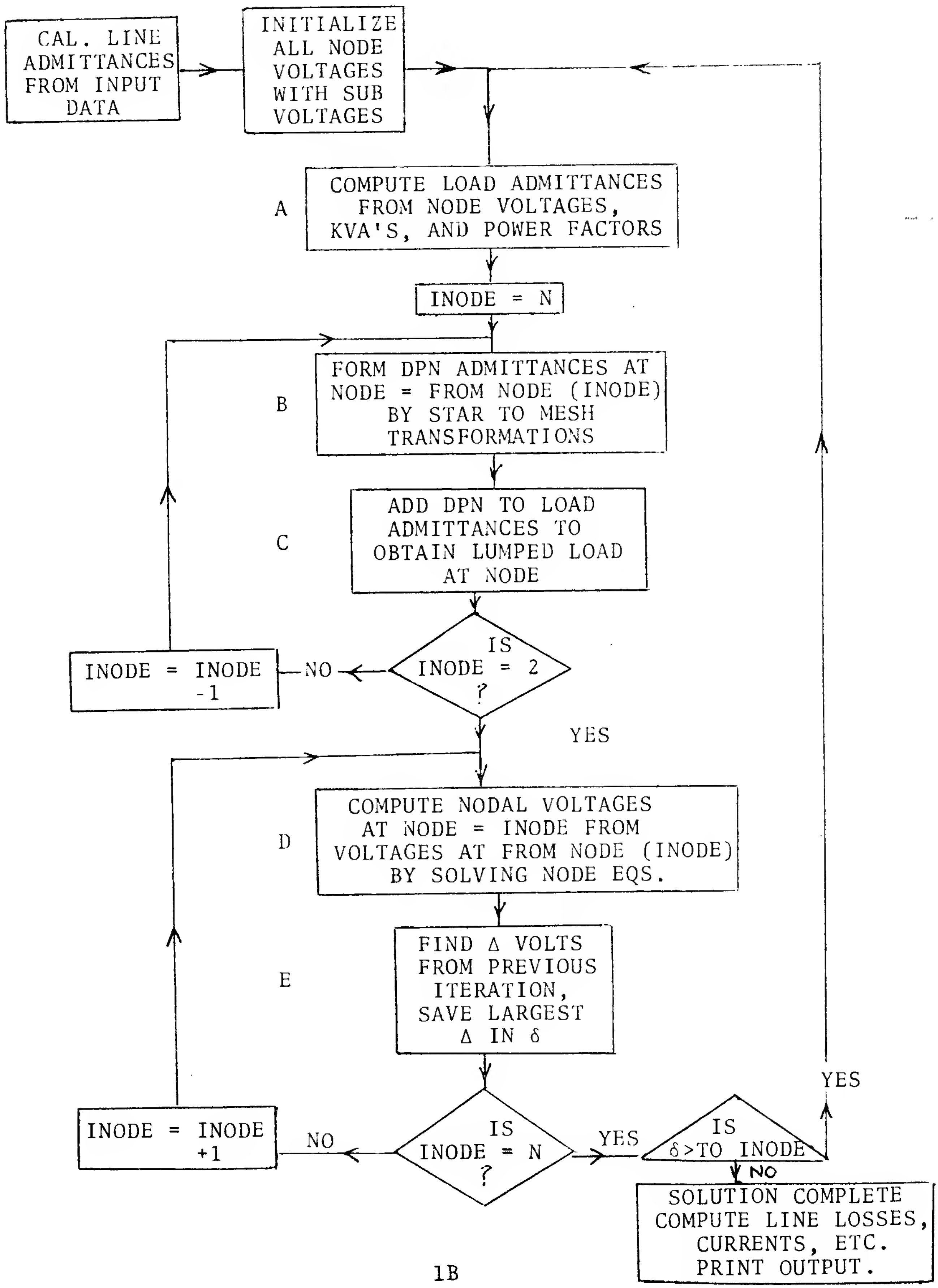
To back up this point, Frank and Jerry have computed with the new program the circuit voltages for the simple 6-node system, given in Figure 7A, Appendix A. Table 1B of Appendix B, page 7B, presents their results compared with voltages given by the former approximate method. Jerry pointed out, "This system isn't too realistic, but it does demonstrate the ideas. The hand calculations using approximate methods give very erroneous results due to the increase in neutral voltage caused by the heavily unbalanced phases."

The computer used was an IBM 360/65, which solved the 6-node problem in 1.47 seconds after six iterations. The typical distribution circuit might have 100 nodes.

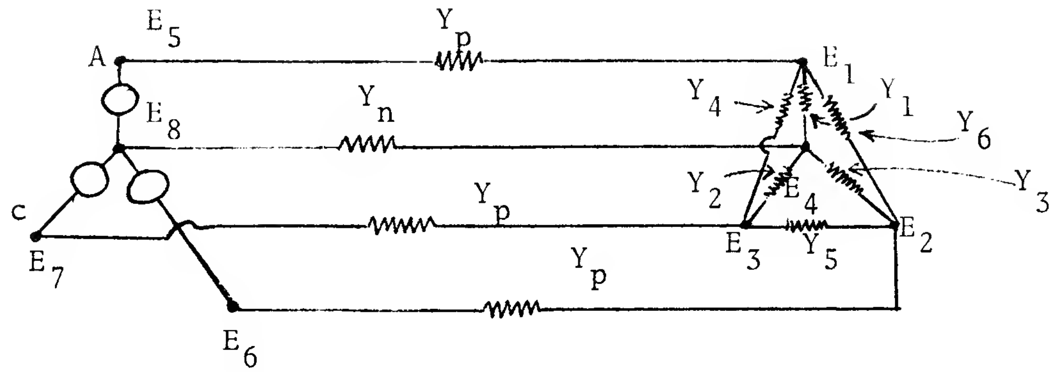
APPENDIX B

EXHIBIT 1B

(By Jerry Guth, Pennsylvania Power and Light Co.)



From Node (I NODE) I -NODE



Equivalent Circuit Node to Node

Node equations for solving E_1, E_2, E_3, E_4 are

$$\begin{bmatrix} Y_1+Y_4+Y_6+Y_p & -Y_6 & -Y_4 & -Y_1 \\ -Y_y & Y_6+Y_3+Y_5+Y_p & -Y_5 & -Y_3 \\ -Y_4 & -Y_5 & Y_4+Y_2+Y_5+Y_p & -Y_2 \\ -Y_1 & -Y_3 & -Y_2 & Y_1+Y_3+Y_2+Y_n \end{bmatrix} \bullet \begin{bmatrix} E_1 \\ E_2 \\ E_3 \\ E_4 \end{bmatrix} = \begin{bmatrix} E_5 Y_p \\ E_6 Y_p \\ E_7 Y_p \\ E_8 Y_n \end{bmatrix}$$

As a matrix equation write

$$YE = I$$

The two four dimensional matrix equations can be written as one eight dimensional equation thus:

$$\begin{bmatrix} Y_x & -Y_y \\ Y_y & Y_x \end{bmatrix} \bullet \begin{bmatrix} E_x \\ E_y \end{bmatrix} = \begin{bmatrix} I_x \\ I_y \end{bmatrix}$$

It is apparent that this system is no longer symmetric as the original complex one was. This rules out the use of special methods such as Cholesky's method in solving the system. In fact Gaussian elimination is as good as any, and it is the method used.

Computations

Block A:

Given KVA loads and corresponding power factors, and node voltages find the admittance. Let

$$\Delta E = \text{voltage across load}$$
$$Y = \text{load admittance}$$
$$I = \text{current thru load}$$

$$P = KW + j\ KVAR = \Delta E \cdot I^* = \text{complex power}$$

$$KVA = |P|$$

so

$$KW = KVA^*\ \text{power factor}$$
$$KVAR = KVA^*\ \ 1 - (\text{power factor})^2$$

$$I - \Delta EY$$

so

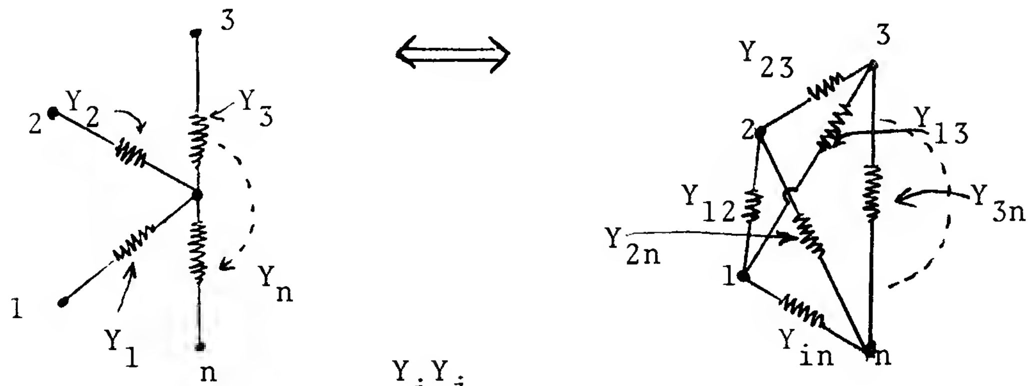
$$P = |\Delta E|^2 Y$$

$$\therefore \quad Y = \frac{KW}{|\Delta E|^2} \pm j\ \frac{KVAR}{|\Delta E|^2}$$

If power factor is negative take negative sign, else positive. Inductive load ⇔ negative power factor.

Blocks B and C:

Given two-node three-phase network find the driving point impedances at the first node looking toward the second, method used is to eliminate all 4 nodes of the second 'node' by 4 star to mesh transformations, one at a time. The general transformation is



$$Y_{ij} = \frac{Y_i Y_j}{\sum_{k=1}^{n} Y_k}$$

This is conveniently done for our case of eight to four node transformation, by the algorithm given in the original paper.

Block D:

Given the circuit of page B, with known admittances and voltages E_5, E_6, E_7 and E_8, find voltages E_1, E_2, E_3 and E_4.

The method used is to solve the system of node equations of page 2B. This gives the voltages immediately, rather than currents as in solving loop equations.

The equations are complex. They could be solved using complex arithmetic by machine, but this is very inefficient. It is best to reduce the problem to real variables. This can be done by enlarging the system of four simultaneous complex linear equations to eight simultaneous real linear ones in the following manner.

Write the complex matrix equations as

$$YE = I \quad \Leftrightarrow \quad \begin{aligned} Y_x E_x - Y_y E_y &= I_x \\ Y_x E_y + Y_y E_x &= I_y \end{aligned}$$

where

$$Y = Y_x + j\, Y_y \text{ etc.}$$

Block E:

The voltages obtained from step D are compared to their values of the previous iteration. Define for the i^{th} node

$$\Delta_i = \max \begin{Bmatrix} (E_1^{n-1} - E_1^n)/E_1^n \\ (E_2^{n-1} - E_2^n)/E_2^n \\ (E_3^{n-1} - E_3^n)/E_3^n \\ \vdots \end{Bmatrix}$$

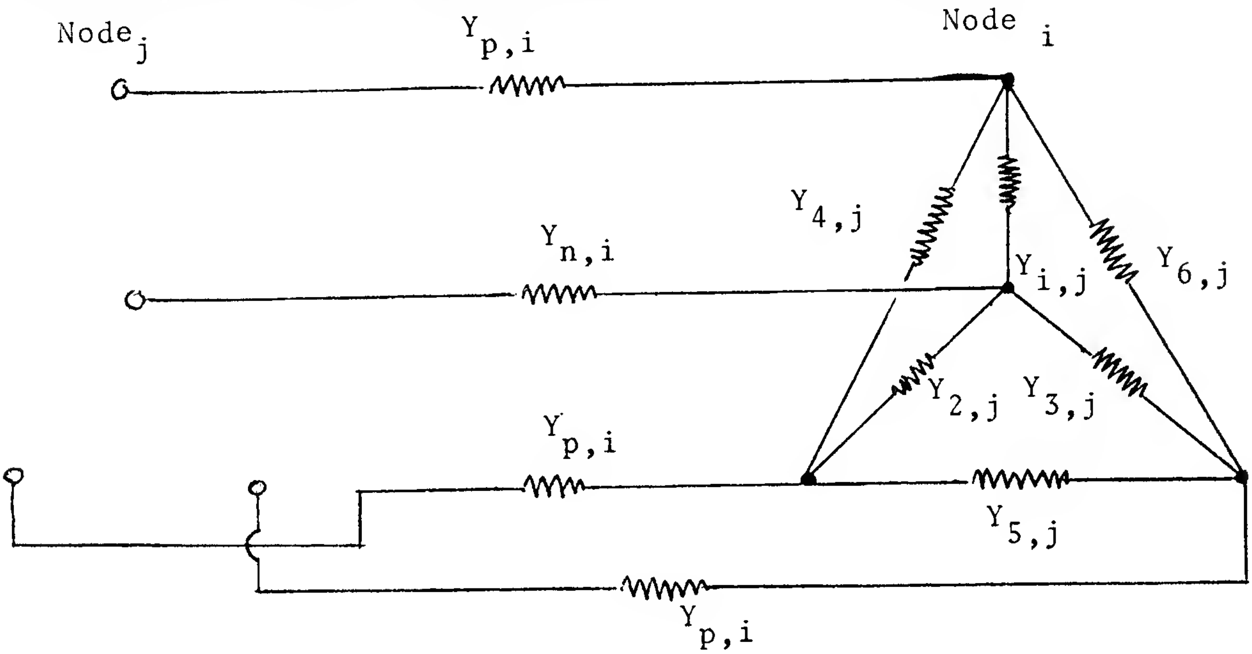
where E_2^n is the value of voltage E_2 obtained in the n^{th} iteration.

When Δ_i has been found for all nodes, then find $\delta = \max \Delta_i$.

If $\delta <$ tolerance the interation is ended. Tolerance is defined by user thru input.

SUPPLEMENT 1B

NETWORK DRIVING POINT ADMITTANCE AND ITERATION METHOD



Form array w(n,m):

n \ m	1	2	3	4	5	6	7
1	$y_{4,i}$	$y_{5,i}$	$y_{3,i}$	0	$y_{4,j}$	$y_{5,j}$	$y_{3,j}$
2	$y_{6,i}$	$y_{2,i}$	0	0	$y_{6,j}$	$y_{2,j}$	0
3	$y_{1,i}$	0	0	0	$y_{1,j}$	0	0
4	$y_{p,i}$	$y_{p,i}$	$y_{p,i}$	$y_{n,i}$	0	0	0

Now for:

r = 1,4
s = 2,4
t = s to 4

find

$$W_{s-1,t-s+1+r} = \frac{(W_{t-s+1,r})\ (W_{t,r})}{\sum_{k=1}^{4} W_{k,r}} + W_{s-1,t-s+1+r}$$

and let

$$Y_{1,j} = W_{3,5} \qquad Y_{4,j} = W_{1,5}$$
$$Y_{2,j} = W_{2,6} \qquad Y_{5,j} = W_{1,6}$$
$$Y_{3,j} = W_{1,7} \qquad Y_{6,j} = W_{2,5}$$

Network DPN Algorithm

These Y's are added to the Y's already existing at node j due to loads on node j or due to some other node which has been "collapsed" onto node j in the same manner.

Repeated application of this method eventually works back to the first node away from the source voltage. Solution of nodal equations here finds node voltages. Then working outward away from this node by repeated solution of nodal equations solves for voltages at each node until the end of the branches are reached.

See Reference 1 of the bibliography for computation of phase and neutral equivalent impedances. The earth return is not considered as zero impedance. Thus all grounded neutral points are not at the same potential.

Y = 0 for an open circuit ($Z = \infty$). This enters zero's into the nodal equation matrices.

TABLE 1B

Results for 6-Node Circuit

Hand (approximate) calculations: with capacitors added

120 v = 1 p.u.; 126 v assumed at A,B,C phases-to-neutral at source, 120° out of phase.

	Node				
Phase	2	3	4	5	6
A	1.037	1.028	1.027	-----	0.962
B	1.037	-----	1.027	1.018	-----
C	1.037	-----	1.027	-----	-----

Case I (computer results - no capacitors)

Phase	2	3	4	5	6
A	1.044	1.040	1.043	-----	1.011
B	1.036	-----	1.017	1.011	-----
C	1.053	-----	1.056	-----	-----

Case II (computer results - with capacitors)

Phase	2	3	4	5	6
A	1.047	1.044	1.052	-----	1.025
B	1.039	-----	1.026	1.022	-----
C	1.050	-----	1.049	-----	-----

EXHIBIT 2B

```
STMT LEVEL NEST
   1            PLSAC1T: PROC OPTIONS(MAIN);
   2     1               DECLARE
                /*--------------------------------------------------------------------
                         SINGLE PRECISION FLOATING POINT STATIC VARIABLES
                ------------------------------------------------------------------*/
                            (V1,V2,V3,V4,PHASE,LODZ,
                             DELTA_E(6,2),WORK(4,7,2),CUR(3),Y(4,4,2),Z(6,8),
                             VOLTS(300,4,2),KVA(300,6),UPNY(300,6,2),PHASEY(300,2),
                             POWER_FACTOR(300,6),NEUTY(300,2),PROD(4,4),
                             PF,SINPF,ESUM,EX,EY,KVAZ,
                             SUMX,SUMY,SUMSQ,
                             WORK1X,WORK1Y,WORK2X,WORK2Y,WORK12X,WORK12Y,PRODX,PRODY,
                             Y1,Y2,Y3,Y4,Y5,Y6,YP,YN,
                             VOLTX,VOLTY,YPX,YPY,SIZE,PIVOT,MULT,MAX,
                             GKM,DIV,TOLERANCE,DIFFERENCE,VIJ,
                             V(8),DPN(6)
                                                                ) STATIC FLOAT,
                         /* DOUBLE PRECISION */
                             SUM                         STATIC    FLOAT(16),
                             /* OTHER  */
                             TEST                        STATIC    FIXED BIN(15),

                /*--------------------------------------------------------------------
                         HALF WORD BINARY INDICIES AND/OR COUNTERS
                ------------------------------------------------------------------*/
                             (I,J,IY,INODE,JNODE,LAST_NODE,R,K,R1,S,SM1,SMP1,M,
                             PI(8),PTI,SAVE_PT,PIVOT_INDEX,
                             FROM_NODE(300),
                              T,TT,IROW,ICOL,IIROW,IPROD,II,III
                                                         ) STATIC    FIXED BIN(15),
                             /* OVERLAYS */
                             LOADY(300,3,2)                 FLOAT   DEFINED UPNY;


   3     1      ITERATE:
                         TEST=0;  /* SET TEST SWITCH FOR CONVERGENCE CRITERION  */


                /*--------------------------------------------------------------------
                         THIS ROUTINE WILL CALCULATE LOAD ADMITTANCES ASSUMING CONSTANT
                         KVA LOADS AT GIVEN POWER FACTORS USING NODE VOLTAGES COMPUTED
                         IN PREVIOUS ITERATIONS
                ------------------------------------------------------------------*/

   4     1               DO INODE=2 TO LAST_NODE;
                /********   CALCULATE VOLTAGE ACROSS Y FROM NODE VOLTAGES           */
   5     1     1             DO I=1 TO 2;
   6     1     2                V1=VOLTS(INODE,1,I);
   7     1     2                V2=VOLTS(INODE,2,I);
   8     1     2                V3=VOLTS(INODE,3,I);
   9     1     2                V4=VOLTS(INODE,4,I);
  10     1     2                DELTA_E(1,I)=V1-V4;
  11     1     2                DELTA_E(2,I)=V3-V4;
  12     1     2                DELTA_E(3,I)=V2-V4;
  13     1     2                DELTA_E(4,I)=V3-V1;
  14     1     2                DELTA_E(5,I)=V3-V2;
```

```
       PLSAE10: PROC OPTIONS(MAIN):

STMT LEVEL NEST

 15     1     2                       DELTA_E(6,I)=V2-V1;
 16     1     2                       END;

                      /********    TRANSFORM KVA AND POWER FACTOR TO LOAD ADMITTANCE          */
 17     1     1                    DO IY=1 TO 6;
 18     1     2                       KVAZ=KVA(INODE,IY);
 19     1     2                       IF KVAZ=0
 20     1     2                          THEN DO;
 21     1     3                             LOADY(INODE,IY,*)=C;
 22     1     3                             GO TO LD1;
 23     1     3                             END;
 24     1     2                       PF=POWER_FACTOR(INODE,IY);
 25     1     2                       SINPF=SQRT(1-PF*PF);
 26     1     2                       IF PF<0
 27     1     2                          THEN DO;
 28     1     3                             SINPF=-SINPF;
 29     1     3                             PF=-PF;
 30     1     3                             END;
 31     1     2                       EX=DELTA_E(IY,1);
 32     1     2                       EY=DELTA_E(IY,2);
 33     1     2                       EX=EX*EX;
 34     1     2                       EY=EY*EY;
 35     1     2                       ESUM=EX+EY;
 36     1     2                       LODZ=KVAZ/ESUM;
 37     1     2                       LOADY(INODE,IY,1)=LODZ*PF;
 38     1     2                       LOADY(INODE,IY,2)=LODZ*SINPF;
 39     1     2       LD1:            END;
 40     1     1                    END;

                      /*------------------------------------------------------------------------
                                THIS SECTION REDUCES 8 NODE CIRCUIT TO 4 NODE EQUIVALENT
                                DRIVING POINT NETWORK THROUGH STAP TO MESH TRANSFORMATIONS.
                      ------------------------------------------------------------------------*/

 41     1                       DO INODE=LAST_NODE TO 2 BY -1;
 42     1     1                    JNODE=FROM_NODE(INODE);
 43     1     1                    WORK=0;     /* FILL WORKING ARRAY FOR ALGORITHM */
 44     1     1                    DO I=1 TO 2;
 45     1     2                       WORK(1,1,I)=DPNY(INODE,4,I);
 46     1     2                       WORK(1.2,I)=DPNY(INODE,5,I);
 47     1     2                       WORK(1,3,I)=DPNY(INODE,3.I);
 48     1     2                       WORK(2,1,I)=DPNY(INODE,6,I);
 49     1     2                       WORK(2,2,I)=DPNY(INODE,2,I);
 50     1     2                       WORK(3,1,I)=DPNY(INODE,1,I);
 51     1     2                       WORK(4,4,I)=NEUTY(INODE,I);
 52     1     2                       PHASE=PHASEY(INODE,I);
 53     1     2                       DO J=1 TO 3;
 54     1     3                          WORK(4,J,I)=PHASE;
 55     1     3                          END;
 56     1     2                       END;

                      /********    START OF ALGORITHM FOR STAR - MESH TRANSFORMATION          */
 57     1     1                    DO R=1 TO 4;
```

```
STMT LEVEL NEST

 59     1    2                DO K=1 TO 4;
 60     1    3                   SUMX=SUMX+ WORK(K,R,1);
 61     1    3                   SUMY=SUMY+ WORK(K,R,2);
 62     1    3                   END;
 63     1    2                SUMSQ=SUMX*SUMX+ SUMY*SUMY;
 64     1    2                R1=R+1;
 65     1    2                DO S=2 TO 4;
 66     1    3                   SM1=S-1;
 67     1    3                   SMR1=S-R1;
 68     1    3                   DO T=S TO 4;
 69     1    4                      WORK1X=WORK(T-SM1,R,1);
 70     1    4                      WORK1Y=WORK(T-SM1,R,2);
 71     1    4                      WORK2X=WORK(T,R,1);
 72     1    4                      WORK2Y=WORK(T,R,2);
 73     1    4                      WORK12X=WORK1X*WORK2X - WORK1Y*WORK2Y;
 74     1    4                      WORK12Y=WORK1X*WORK2Y + WORK1Y*WORK2X;
 75     1    4                      PRODX=(WORK12X*SUMX + WORK12Y*SUMY)/SUMSQ;
 76     1    4                      PRODY=(WORK12Y*SUMX - WORK12X*SUMY)/SUMSQ;
 77     1    4                      TT=T-SMR1;
 78     1    4                      WORK(SM1,TT,1)=PRODX+ WORK(SM1,TT,1);
 79     1    4                      WORK(SM1,TT,2)=PRODY+ WORK(SM1,TT,2);
 80     1    4                      END;
 81     1    3                   END;
 82     1    2                END;

                 /********  ADD DPN ADMITTANCES TO LOAD ADMITTANCES OF JNODE TO        */
                           GET THE TOTAL DPN Y FOR JNODE
 83     1    1             DO I=1 TO 2;
 84     1    2                DPN(1)= WORK(3,5,I);
 85     1    2                DPN(2)= WORK(2,6,I);
 86     1    2                DPN(3)= WORK(1,7,I);
 87     1    2                DPN(4)= WORK(1,5,I);
 88     1    2                DPN(5)= WORK(1,6,I);
 89     1    2                DPN(6)= WORK(2,5,I);
 90     1    2                DO IY=1 TO 6;
 91     1    3                   DPNY(JNODE,IY,I)=LOADY(JNODE,IY,I) + DPN(IY);
 92     1    3                   END;
 93     1    2                END;
 94     1    1             END;

                 /*-----------------------------------------------------------------------
                        THIS SECTION CALCULATES NODE VOLTAGES AT THE TO-NODE GIVEN
                        THE VOLTAGES AT THE FROM NODE, THE LINE ADMITTANCES BETWEEN
                        THE NODES, AND THE DPN ADMITTANCES AT THE TO-NODE.  THE
                        SOLUTION IS OBTAINED BY SOLVING 8 SIMULTANEOUS REAL LINEAR
                        EQS BY GAUSSIAN ELIMINATION WITH PARTIAL PIVOTING.  THE EQS.
                        ORIGINATE AS 4 COMPLEX NODE EQS   YE=I .
                 ------------------------------------------------------------------------*/

 95     1                   DO INODE=2 TO LAST_NODE;
 96     1    1                 JNODE=FROM_NODE(INODE);
 97     1    1                 DO I=1 TO 2;   /* RE AND IMAG PARTS */
 98     1    2                    Y1=DPNY(INODE,1,I);
 99     1    2                    Y2=DPNY(INODE,2,I);
```

```
PLSAE10: PROC OPTIONS(MAIN);

STMT LEVEL NEST

100    1    2                    Y3=DPNY(INODE,3,I);
101    1    2                    Y4=DPNY(INODE,4,I);
102    1    2                    Y5=DPNY(INODE,5,I);
103    1    2                    Y6=DPNY(INODE,6,I);
104    1    2                    YP=PHASEY(INODE,I);
105    1    2                    YN=NEUTY(INODE,I);

                /********FORM NODAL ADMITTANCE MATRIX                                          */
106    1    2                    Y(1,1,I)=Y1+Y4+Y6+YP;
107    1    2                    Y(2,2,I)=Y6+Y3+Y5+YP;
108    1    2                    Y(3,3,I)=Y4+Y2+Y5+YP;
109    1    2                    Y(4,4,I)=Y1+Y3+Y2+YN;
110    1    2                    Y(1,2,I),Y(2,1,I)=-Y6;
111    1    2                    Y(1,3,I),Y(3,1,I)=-Y4;
112    1    2                    Y(1,4,I),Y(4,1,I)=-Y1;
113    1    2                    Y(2,3,I),Y(3,2,I)=-Y5;
114    1    2                    Y(4,2,I),Y(2,4,I)=-Y3;
115    1    2                    Y(4,3,I),Y(3,4,I)=-Y2;
116    1    2                    END;

                /********  FORM CURRENT VECTOR FOR EQ YE=I                                     */
117    1    1                YPX=PHASEY(INODE,1);
118    1    1                YPY=PHASEY(INODE,2);
119    1    1                DO IROW=1 TO 4;
120    1    2                    VOLTX=VOLTS(JNODE,IROW,1);
121    1    2                    VOLTY=VOLTS(JNODE,IROW,2);
122    1    2                    CUR(IROW)=VOLTX*YPX-VOLTY*YPY;
123    1    2                    CUR(IROW+4)=VOLTX*YPY + VOLTY*YPX;
124    1    2                    END;

                /******** THE 4 SIMULTANEOUS COMPLEX LINEAR EQS. ARE EQUIVALENT TO
                          8 SIMULTANEOUS REAL LINEAR EQS. THESE ARE FORMED NEXT.
                          THEY ARE   YE=I    <=>  |YX  -YY||EX| - |IX|
                                                  |YY   YX||EY| - |IY|                 */

                /******** FORM 8X8 MATRIX EQUIVALENT OF 4  COMPLEX  EQS.                       */
125    1    1                DO I=1 TO 4;
126    1    2                    DO J=1 TO 4;
127    1    3                        Z(I,J),Z(I+4,J+4)=Y(I,J,1);
128    1    3                        Z(I+4,J)=Y(I,J,2);
129    1    3                        Z(I,J+4)=-Y(I,J,2);
130    1    3                        END;
131    1    2                    END;

                /*----------------------------------------------------------------------
                         THIS ROUTINE DOES GAUSSIAN ELIMINATION WITH PARTIAL PIVOTING
                         TO SOLVE THE 8X8 SYSTEM.
                ----------------------------------------------------------------------*/

                /******** LU DECOMPOSITION                                                     */
132    1    1                DO I=1 TO 8;  /* INITIALIZE POINTER FOR INTERCHANGING ROW*/
133    1    2                    PT(I)=I;
```

```
STMT LEVEL NEST

135    1    1              DO K=1 TO 7;
136    1    2                 MAX=0;
137    1    2                 DO I=K TO 8;  /* SEARCH FOR MAX ELEMENT IN COL */
138    1    3                    SIZE=ABS( Z(PT(I),K) );
139    1    3                    IF SIZE>MAX
140    1    3                       THEN DO;
141    1    4                         MAX=SIZE;
142    1    4                         PIVOT_INDEX=I;
143    1    4                         END;
144    1    3                    END;
145    1    2                 IF PIVOT_INDEX¬=K
146    1    2                    THEN DO;
147    1    3                       SAVE_PT=PT(K);
148    1    3                       PT(K)=PT(PIVOT_INDEX);
149    1    3                       PT(PIVOT_INDEX)=SAVE_PT;
150    1    3                       END;
151    1    2                 PIVOT=Z(PT(K),K);
152    1    2                 DO I=K+1 TO 8;
153    1    3                    PTI=PT(I);
154    1    3                    Z(PTI,K),MULT =Z(PTI,K)/PIVOT;
155    1    3                    IF MULT¬=0
156    1    3                       THEN DO;
157    1    4                         DO J=K+1 TO 8;
158    1    5                            Z(PTI,J)=Z(PTI,J)-MULT*Z(PT(K),J);
159    1    5                            END;
160    1    4                         END;
161    1    3                    END;
162    1    2                 END;

                /******** FORWARD ELIMINATION                                          */
163    1    1              DO I=1 TO 8;
164    1    2                 SUM=0;
165    1    2                 PTI=PT(I);
166    1    2                 DO J=1 TO I-1;
167    1    3                    SUM=SUM+ MULTIPLY(Z(PTI,J),V(J),16);
168    1    3                    END;
169    1    2                 V(I)=CUR(PTI)-SUM;
170    1    2                 END;

                /******** BACKWARDS SUBSTITUTION                                       */
171    1    1              DO I=8 TO 1 BY -1;
172    1    2                 SUM=0;
173    1    2                 PTI=PT(I);
174    1    2                 DO J=I+1 TO 8;
175    1    3                    SUM=SUM+ MULTIPLY(Z(PTI,J),V(J),16);
176    1    3                    END;
177    1    2                 V(I)=(V(I)-SUM)/Z(PTI,I);
178    1    2                 END;


179    1    1              IF TEST¬=0
180    1    1                 THEN GO TO NEWV;
                /********  TEST FOR MAGNITUDE OF CHANGE FROM PREVIOUS ITERATION     */
```

```
PLSAE10: PROC OPTIONS(MAIN);

STMT LEVEL NEST

181      1     1                     DO I=1 TO 2;
182      1     2                        IF I=1
183      1     2                           THEN IJ=0;
184      1     2                           ELSE IJ=4;
185      1     2                        DO J=1 TO 3;
186      1     3                           VIJ=V(J+IJ);
187      1     3                           DIFFERENCE=ABS( (VOLTS(INODE,J,I)-VIJ)/VIJ);
188      1     3                           IF DIFFERENCE>TOLERANCE
189      1     3                              THEN DO;
190      1     4                                TEST=1;
191      1     4                                GO TO NEWV;
192      1     4                                END;
193      1     3                           END;
194      1     2                        END;
195      1     1         NEWV:
                                        DO I=1 TO 4;
196      1     2                           VOLTS(INODE,I,1)=V(I);
197      1     2                           VOLTS(INODE,I,2)=V(I+4);
198      1     2                           END;
199      1     1                     END;


200      1                        IF TEST=1
201      1                           THEN GO TO ITERATE;
202      1                        END PLSAE10;
```

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INSTRUCTOR'S GUIDE

CASE TITLE: Radial Load Flow Problems in Distribution Circuit Analysis

RELATED UNDERGRADUATE CURRICULUM AREAS:

1. Engineering analysis and design methods (electrical engineering and computer science orientation).

2. Electric circuit analysis (Node-voltage method, y-Δ transformations, practical application of power factor correction concept).

3. Computer science

   a. Numerical--iteration procedure, Gaussian elimination, complex numbers.

   b. Non-numerical--traversing a tree data structure in pre and end-order.

4. Power systems analysis and control.

SYNOPSIS: Calculation of the voltage drop from one point to the next point along a three-phase, low-voltage distribution circuit's transmission line can be done by approximate means fairly accurately. However, in some cases, exact calculation of drops was found desirable by engineers at the Pennsylvania Power and Light Company. The problem's formulation introduces a large number of simultaneous nonlinear algebraic equations which have complex co-efficients--an obvious instance where the digital computer and numerical methods can be used to advantage.

In this case, an electrical engineer and a scientific programmer work together to develop a computer program which depends on electric circuit equations and an iterative procedure to solve them. The tree-like or radial nature of the typical distribution circuit required devising a way to sequence the computations properly. After the program was developed, a test was run on a simple circuit which could also be hand-calculated, and it was shown that computer results are quite different from the approximate results which otherwise would have had to be accepted.

The simple concepts behind the iterative process and the computation sequence problem are easily shown to students without cluttering up the basic ideas with too much algebra. An interesting assignment in computer processing can thus be devised.

The case study also shows how locating capacitors at various points indicated by computation can improve voltage regulation and lessen thermal losses in lines and transformers--an illustration of the utilities' concern with avoiding waste of electric energy.

QUESTIONS FOR THOUGHT AND DISCUSSION

After reading Part A:

1. What are the primary reasons for controlling voltage swings between no-load and full-load conditions on a distribution circuit?

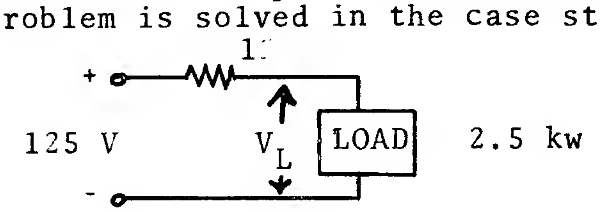
2. What part did a literature search play in the course of the project?

3. A decision was made by the engineer on the extent to which he would explain the problem to the scientific programmer who was not an electrical engineer. Discuss this decision.

4. Clearly explain the concept of power-factor correction. Why might a large power system find it better to operate at 0.85 or 0.90 pf rather than nearer unity?

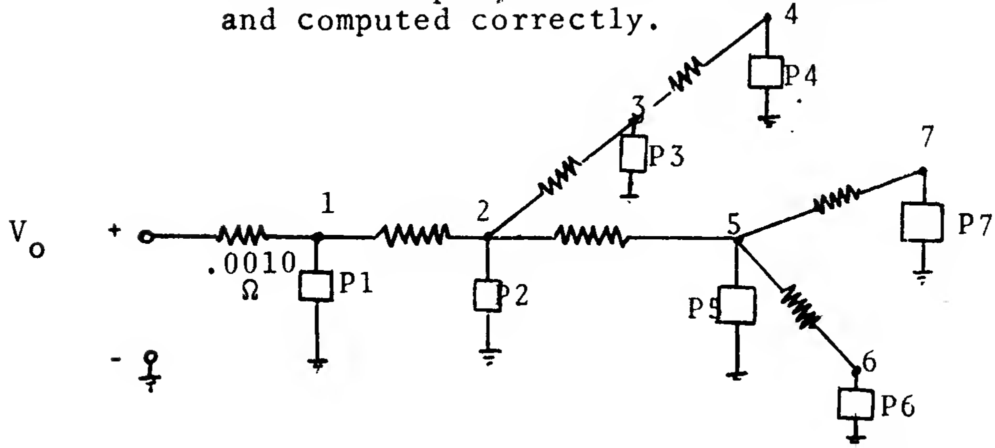
After reading Part B:

1. Given the problem below, solve for the two possible load voltages. What possible difficulty is suggested with respect to the way the more complex problem is solved in the case study? Discuss.



Hint: The question raises the problem always related to an iterative procedure--that of convergence. If the iteration procedure used in the case study is used here, the assumed starting value for V_L will lead to convergence to one of the two possible V_L values.

2. Discuss the programming of the simpler DC problem below with the aim of making a general program, i.e., if a different set of cards as listed is used for input, the new circuit would be traversed and computed correctly.



From Node	To Node	R(Ω)	Load at To Node (ω)
0	1	.0010	2.5
1	2	.0020	5.0
2	3	.0005	10.0
3	4	.0015	5.0
2	5	.0020	6.0
5	6	.0010	10.0
5	7	.0010	8.0

Answers after 10 iterations (with 'flat' start) on IBM-360/65:

V0 = 1.0000
V1 = .9412
V2 = .8290
V3 = .8198
V4 = .8105
V5 = .7655
V6 = .7522
V7 = .7549